The Seven-Dimensional Spacetime in the Planck Vacuum Theory and the Structure of the Electron and Proton Cores

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Abstract—The electron and proton cores in the Planck vacuum (PV) theory are recognizable elementary particles that obey the manifestly covariant Dirac equation and that are coupled to the PV state. (This statement and similar statements to follow also apply to the electron-core and proton-core antiparticles.) This paper derives the corresponding 2x1 spinor-wavefunction equations for these cores, leading to a 7-dimensional spacetime that consists of two 4-dimensional spacetimes, the result of a bifurcated PV state. Both the electron and proton cores contain structure, where the electron-core structure is orders of magnitude smaller than the proton-core structure. The core structure is easily recognized in the calculations, as is particle-antiparticle annihilation. The difference between the electron and proton cores and their corresponding electron and proton particles is that the latter contain radiative corrections.

Index Terms—Fundamental Physics, Spacetime, Dirac Equation, Spinor Wavefunction, Particle-Antiparticle Annihilation, Planck Vacuum, Vacuum Structure.

I. INTRODUCTION

The PV theory is a quasi-particle theory rather than a quantum field theory, where the electron and proton cores are particle-like objects (consisting of a massive point charge) that are coupled to the invisible PV state. Using this model, the wavefunctions for these cores are derived from the manifestly covariant Dirac equation, leading to two 2x1 spinor wavefunctions. These 2x1 reduced spinors and the four equations describing them suggest that two of the equations belong to the observable 4-dimensional spacetime, and two of the equations belong to a separate 4-dimensional unobserved spacetime. The reason for the two spacetimes is to separate the oppositely charged branches within the PV state.

The theoretical foundation [1] [2] [3] of the PV theory rests upon the unification of the Einstein, Newton, and Coulomb superfices:

\[
\frac{e^4}{G} = \frac{m_e c^2}{r_e} = \frac{m_p^2 G}{r_p} = \frac{e^2}{r_e^2} = \frac{e^2}{r_p^2}
\]

where the ratio \(e^4/G\) is the curvature superforce that appears in the Einstein field equations. \(G\) is Newton’s gravitational constant, \(c\) is the speed of light, \(m_e\) and \(r_e\) are the Planck mass and length respectively [4, p.1234], and \(e_+\) is the massless bare (or coupling) charge. The fine structure constant is given by the ratio \(\alpha \equiv e^2/e_+^2\), where \(e\) is the observed electronic charge.

The two particle/PV coupling forces

\[
F_e(r) = \frac{e_+^2}{r^2} - \frac{m_e c^2}{r} \quad \text{and} \quad F_p(r) = \frac{e_+^2}{r^2} - \frac{m_p c^2}{r}
\]

(2)

where the ratio \(e^4/G\) is the curvature superforce that appears in the Einstein field equations. \(G\) is Newton’s gravitational constant, \(c\) is the speed of light, \(m_e\) and \(r_e\) are the Planck mass and length respectively [4, p.1234], and \(e_+\) is the massless bare (or coupling) charge. The fine structure constant is given by the ratio \(\alpha \equiv e^2/e_+^2\), where \(e\) is the observed electronic charge.

The electron core \((-e_+, m_e)\) and the proton core \((e_+, m_p)\) exert on the invisible PV state; along with their coupling constants

\[F_e(r_e) = 0 \quad \text{and} \quad F_p(r_p) = 0\]

(3)

and the resulting Compton radii

\[r_e = \frac{e_+^2}{m_e c^2} \quad \text{and} \quad r_p = \frac{e_+^2}{m_p c^2}\]

(4)

lead to the important string of Compton relations

\[r_e m_e c^2 = r_p m_p c^2 = e_+^2 = r_e m_e c^2 \quad (= c\hbar)\]

(5)

for the electron and proton cores, where \(\hbar\) is the reduced Planck constant. The Compton relation to the right of \(e_+^2\) comes from equating the Einstein and Coulomb superfices from (1). To reiterate, the equations in (2) represent the forces the free electron and proton cores exert on the invisible PV space, a continuum that is itself pervaded by a degenerate collection of Planck particle cores \((\pm e_+, m_e)\) [5], leading to a bifurcated vacuum state with one positive branch \((e_+, m_e)\) and one negative branch \((-e_+, m_e)\). The Lorentz invariance of the coupling constants in (3) leads to the energy and momentum operators of the quantum theory [5] [6].

Section 2 derives the core equations; Section 3 develops the reduced spinor equations; and Section 4 separates the four resulting equations into two separate spacetimes of 4-dimensions each, leading to a combined 7-dimensional spacetime.

This is the third paper in a series that began with the paper “The Anomalous Magnetic Moment of the Electron and Proton Cores According to the Planck Vacuum Theory” [6], that seeks to discover a model for the electron and proton particles that is “beyond the standard model”. The present paper and the magnetic moment paper constitute a complete and definitive description of the electron and proton particles.

II. DIRAC EQUATION FOR THE CORES

The Dirac particles defined in this paper are the electron and proton cores and their antiparticles \((e_+, m_e)\) and \((-e_+, m_p)\), where the following Dirac equations are assumed to result from the forces in (2) operating on the PV state. This assumption follows from the appearance of \(e_+^2\) and \(m\) in the following equations, where \(m\) is the electron or proton rest mass.
Using (5), the manifestly covariant form [7, p.90] [Appendix A] of the Dirac equation for the electron and proton cores can be expressed as:
\[
(\ii h c^2 \gamma^\mu \frac{\partial}{\partial x^\mu} - mc^2) \psi = \left( \frac{\ii e^2}{c} \gamma^\mu \frac{\partial}{\partial x^\mu} - mc^2 \right) \psi = 0
\]  
(6)
where one of the charges in \( e^2 \) belongs to the free electron or proton cores, and the other to any one of the Planck-particle cores within the degenerate PV state.

Using Appendix A, equation (6) can be expanded to
\[
i \frac{e^2}{c} \left[ \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \frac{\partial}{\partial x^0} + \gamma^0 \alpha_j \frac{\partial}{\partial x^j} \right] \psi = mc \left[ \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix} \right] \psi
\]  
(7)
where \( \psi \) is the 4x1 spinor solution, and where the second term in (7) is summed over \( j = 1, 2, 3 \).

Replacing the 4x1 spinor \( \psi \) by the column vector
\[
\psi = \begin{pmatrix} u \\ v \end{pmatrix}
\]  
(8)
where \( u \) and \( v \) are the 2x1 reduced spinors, leads to the split solution for (6)
\[
i \frac{e^2}{c} \left[ \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \frac{\partial}{\partial x^0} \left( \begin{pmatrix} u \\ v \end{pmatrix} \right) + \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \alpha_j \frac{\partial}{\partial x^j} \left( \begin{pmatrix} u \\ v \end{pmatrix} \right) \right] = mc \left[ \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix} \right] \begin{pmatrix} u \\ v \end{pmatrix}
\]  
(9)
Evaluating the second term of (9) in Appendix B leads to
\[
i \frac{e^2}{c} \left[ \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \frac{\partial}{\partial x^0} \left( \begin{pmatrix} u \\ v \end{pmatrix} \right) + \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \sigma_j \frac{\partial}{\partial x^j} \left( \begin{pmatrix} u \\ v \end{pmatrix} \right) \right] = mc \left[ \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix} \right] \begin{pmatrix} u \\ v \end{pmatrix}
\]  
(10)
where it is noted that the second u-v vector in (9) is now the v-u vector in (10).

Equation (10) reduces to the following two equations
\[
i \frac{e^2}{c} \left( \frac{\partial}{\partial x^0} + \sigma_j \frac{\partial}{\partial x^j} \right) = mcu
\]  
(11)
and
\[
i \frac{e^2}{c} \left( \frac{\partial}{\partial x^0} + \sigma_j \frac{\partial}{\partial x^j} \right) = mcv
\]  
(12)
for the \( u \) and \( v \) reduced spinors. It is important to note that equations (11) and (12) are coupled together by their second terms on the left side of the equations. This coupling is the first hint that the two particles represented by (11) and (12) are related, leading to the idea of the particle-antiparticle pair.

The ratio \( e^2/c \) in (11) and (12) is the spin coefficient [6], where
\[
\sqrt{S} = \frac{e^2}{c} \sigma^2 \quad \rightarrow \quad \frac{e^2}{c} \sigma_j \frac{\partial}{\partial x^j}
\]  
(13)
is the relativistic spin of the electron and proton cores. The Pauli spin vector is \( \sigma \). The second expression in (13) is the scalar product of \( \sqrt{S} \) with the gradient operator \( \partial/\partial x^j \); that is, the PV gradient \( \partial/\partial x^j \) in the \( j \)th direction weighted by the relativistic spin in that direction.

### III. Reduced Spinor Equations
In what follows, the notation \( [(e_1)(e_2)] \) is interpreted in the following manner: \( e_1 \) is the free-space core charge, plus or minus; and \( e_2 \) represents the charges within the PV branches \( (\pm e_*, m_*) \). Using this notation, (11) and (12) are expanded, leading to the sequence of equations
\[
i \frac{[-(e_*(-e_*)]}{c} \left( \frac{\partial}{\partial x^0} + \sigma_j \frac{\partial}{\partial x^j} \right) = mcu
\]  
(14)
\[
i \frac{[(e_*)(-e_*)]}{c} \left( \frac{\partial}{\partial x^0} + \sigma_j \frac{\partial}{\partial x^j} \right) = mcv
\]  
(15)
for the electron and positron, and
\[
i \frac{[-(e_*)(e_*)]}{c} \left( \frac{\partial}{\partial x^0} + \sigma_j \frac{\partial}{\partial x^j} \right) = m_pcu
\]  
(16)
\[
i \frac{[(e_*)(e_*)]}{c} \left( \frac{\partial}{\partial x^0} + \sigma_j \frac{\partial}{\partial x^j} \right) = m_pcv
\]  
(17)
for the proton and antiproton. To read these core equations, taking (14) as an example: the first \( -(e_*) \) and the mass \( m_e \) identifies (14) as the electron core equation, while the second \( -(e_*) \) indicates that the core is coupled to the negative PV branch \( -(e_*, m_*) \). In summary, these four equations, from top to bottom, represent the electron, positron, proton, and antiproton cores respectively.

### IV. Spacetime and Dimensionality
Equations (11) through (17) are 4-dimensional equations (one \( x^0 \) and three \( x^j \)'s); thus the spacetimes associated with (11)–(17) are 4-dimensional. However, (14) and (16) refer to the well-known observable spacetime, while (15) and (17) refer to an unobserved spacetime. Furthermore, in order to separate the positive PV branch \( (e_*, m_*) \) from the negative branch \( -(e_*, m_*) \), or vice versa, the overall dimensionality of the spacetime must be seven dimensional (one \( x^0 \) and six \( x^j \)’s).

The particle cores (14) and (16) belong to the observed spacetime, while the antiparticle cores (15) and (17) belong to the unobserved spacetime.

### V. Comments and Conclusions
At this point in the evolution of the PV theory, the vacuum state is assumed to be a quasi-continuum: a continuum that is pervaded by a degenerate collection of Planck-particle cores \( \pm (e_*, m_*) \). This pervasion leads to a bifurcated vacuum state of two branches: one branch associated with the positive cores \( (e_*, m_*) \) and one branch associated with the negative cores \( -(e_*, m_*) \).

The reduced spinor equations (14)–(17) for the Dirac equation show the following properties: the electron cores \( \pm (e_*, m_*) \) are coupled to the negative PV branch, while the proton cores \( \pm (e_*, m_p) \) are coupled to the positive PV branch; the electron and proton cores satisfy equations (14) and (16), while their corresponding antiparticle cores \( (e_*, m_*) \) and \( -(e_*, m_p) \) satisfy equations (15) and (17) respectively.

The \( -e^2/c \) in (15) implies that the electron anticontact is a structured particle [8]—but the second coupling-term in (15)
implies that the electron core must then also be a structured particle. The same conclusions apply to the proton core and its antiparticle. That is, both the electron and proton cores and their antiparticles are structured particles. However, the electron-to-proton ratio of the Coulomb forces

\[ \frac{e_e^2/r_e^2}{e_p^2/r_p^2} = \frac{r_p^2}{r_e^2} \approx \frac{1}{1836^2} \]  \hspace{1cm} (18)

from the particle/PV coupling forces (2) implies that the electron-core and its antiparticle structure are orders-of-magnitude smaller than those of the proton-core and its antiparticle. This last conclusion explains why the electron and positron show little to no particle structure experimentally.

In the PV theory of the electron and proton cores, their structure [8] is due to the Coulomb attractions \([e_e](-e_e)\) and \([-e_e](-e_e)\) in (15) and (17) for the negative and positive charges of the PV negative and positive branches respectively. These attractions create a small spherical collar around the core Compton-radii \(r_e\) and \(r_p\) respectively.

Finally, the core equations (11) and (12) are assumed to represent a particle-antiparticle pair respectively. Thus, the superposition of the two equations (adding their separate components) should lead to particle-antiparticle annihilation (PAA):

\[ (11) \oplus (12) = \]

\[ i \left[ \begin{array}{c} 0 \\ \frac{e}{c} \end{array} \right] \left[ \begin{array}{cc} \partial(u + v) \\ \partial x^0 + \sigma_j \partial(u + v) \end{array} \right] = mc(u + v) \]  \hspace{1cm} (19)

with the simple solution

\[ u + v = \hat{0} \]  \hspace{1cm} (20)

where \(\hat{0}\) is the null 2x1 reduced spinor. This result constitutes particle-antiparticle annihilation in the PV theory. Using (16) and (17) as an example, the PAA equation for the proton-antiproton

\[ (16) \oplus (17) = \]

\[ i \left[ \begin{array}{c} 0 \\ 0 \end{array} \right] \left[ \begin{array}{cc} \partial(u + v) \\ \partial x^0 + \sigma_j \partial(u + v) \end{array} \right] = m_p c(u + v) \]  \hspace{1cm} (21)

shows, in particular, that the annihilation process is a charge-annihilation process as displayed by the (0) at the left in the equation.

**APPENDIX A**

**The \( \gamma \) and \( \beta \) Matrices**

The 4x4 \( \gamma \), \( \beta \), and \( \alpha \) matrices used in the Dirac theory are defined here: where [7, p.91]

\[ \gamma^0 \equiv \beta = \left( \begin{array}{cc} I & 0 \\ 0 & -I \end{array} \right) \]  \hspace{1cm} (A1)

and \((j = 1, 2, 3)\)

\[ \gamma^j \equiv \beta \alpha_j = \left( \begin{array}{cc} 0 & \sigma_j \\ -\sigma_j & 0 \end{array} \right) \]  \hspace{1cm} (A2)

and where \( I \) is the 2x2 unit matrix and

\[ \alpha_j = \left( \begin{array}{cc} 0 & \sigma_j \\ \sigma_j & 0 \end{array} \right) \]  \hspace{1cm} (A3)

where the \( \sigma_j \) are the 2x2 Pauli spin matrices

\[ \sigma_1 = \left( \begin{array}{cc} 0 & 1 \\ 1 & 0 \end{array} \right), \sigma_2 = \left( \begin{array}{cc} 0 & -i \\ i & 0 \end{array} \right), \sigma_3 = \left( \begin{array}{cc} 1 & 0 \\ 0 & -1 \end{array} \right) \]  \hspace{1cm} (A4)

and \( \alpha = (\alpha_1, \alpha_2, \alpha_3) \). The zeros in (A1)–(A3) and (A5) are 2x2 null matrices.

The \( mc \) in (6) represents the 4x4 matrix

\[ mc = \left( \begin{array}{cc} I & 0 \\ 0 & I \end{array} \right) \]  \hspace{1cm} (A5)

where \( \psi \) is the 4x1 spinor matrix. The zero on the right side of (6) represents the 4x4 null matrix and the zeros in (7) represent 2x2 null matrices.

The coordinates \( x^\mu \) are

\[ x^\mu = (x^0, x^1, x^2, x^3) \]  \hspace{1cm} (A6)

where \( x^0 \equiv ct. \)

**APPENDIX B: Spatial Gradient**

From the second term in (9) and Appendix A:

\[ \gamma^0 \alpha_j \frac{\partial}{\partial x^j} \left( \begin{array}{c} u \\ v \end{array} \right) = \gamma^0 \left( \begin{array}{c} 0 \\ \sigma_j \end{array} \right) \frac{\partial}{\partial x^j} \left( \begin{array}{c} u \\ v \end{array} \right) \]

\[ = \gamma^0 \frac{\partial}{\partial x^j} \left( \begin{array}{c} 0 \\ \sigma_j \end{array} \right) \left( \begin{array}{c} u \\ v \end{array} \right) = \gamma^0 \frac{\partial}{\partial x^j} \left( \begin{array}{c} \sigma_j u \\ \sigma_j v \end{array} \right) \]

\[ = \gamma^0 \sigma_j \frac{\partial}{\partial x^j} \left( \begin{array}{c} v \\ u \end{array} \right) = \left( \begin{array}{cc} I & 0 \\ 0 & -I \end{array} \right) \sigma_j \frac{\partial}{\partial x^j} \left( \begin{array}{c} v \\ u \end{array} \right) \]

where the first \( v-u \) vector at the upper left is replaced by the final \( v-u \) vector at the lower right.

**REFERENCES**


[8] W.C. Daywitt, The Structured Proton and the Structureless Electron as Viewed in the Planck Vacuum Theory, Progress in Physics, v.11, Issue 2 (April), 117, 2015. Note: at this point in the development of the PV theory, the author believed the commonly accepted axiom that the electron particle is structureless.