A Heuristic Model for the Active Galactic Nucleus Based on the Planck Vacuum Theory

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The standard explanation for an active galactic nucleus (AGN) is a "central engine" consisting of a hot accretion disk surrounding a supermassive black hole [1, p. 32]. Energy is generated by the gravitational infall of material which is heated to high temperatures in this dissipative accretion disk. What follows is an alternative model for the AGN based on the Planck vacuum (PV) theory [2, Appendix], where both the energy of the AGN and its variable luminosity are explained in terms of a variable photon flux emanating from the PV.

The Einstein field equation

$$\frac{G_{\mu\nu}/6}{1/r_*^2} = \frac{T_{\mu\nu}}{\rho_* c^2} \qquad (1)$$

is probably invalid in much of the region of interest $(0.5 < n_r < 1)$ to the AGN modeling process, especially as n_r gets closer to unity [2]. Ignoring this concern, though, there is a non-black-hole Schwarzschild line element for an extended mass [3, 4] available for consideration. Unfortunately, this incompressible-fluid model is incompatible with the PV theory (see Appendix B). The following calculations provide a rough heuristic way around these modeling problems.

The expression to be used to estimate the mass of an AGN can be derived from the relation between a spherical mass m and its mass density ρ_0

$$m = \frac{4\pi r^3}{3}\,\rho_0 = \frac{8\pi}{6}\,\rho_0 r^3 \qquad (2)$$

where $r\,(\leqslant r_0)$ is the radius of the sphere and ρ_0 is assumed to be constant. This can be expressed as

$$S_r = \frac{mc^2}{r} = \frac{8\pi}{6}\,\frac{\rho_0 c^2}{1/r^2} \qquad (3)$$

in terms of the curvature stress S_r exerted on the PV at the mass' surface. The maximum stress S_* that can be exerted on the PV is given by the first ratio in

$$S_* = \frac{m_* c^2}{r_*} = \frac{8\pi}{6}\,\frac{\rho_* c^2}{1/r_*^2} \qquad (4)$$

which can be transformed to the second ratio by recognizing $\rho_* = m_*/(4\pi r_*^3/3)$ as the mass density of the individual PPs making up the degenerate PV. Dividing equation (3) by (4) leads to

$$\frac{n_r(1/r^2)}{1/r_*^2} = \frac{\rho_0 c^2}{\rho_* c^2} \qquad (5)$$

where the n-ratio

$$n_r = \frac{S_r}{S_*} = \frac{mc^2/r}{m_* c^2/r_*} < 1 \qquad (6)$$

is the relative stress exerted by m. The curvature stress in (3) is infinite if r is allowed to vanish, but the PV theory restricts r to $r > r_*$ [2]. The surface of the AGN is at $r = r_0$ where $m = m_0$.

As an aside, it is interesting to note that the result in (5) can be made to resemble the Einstein equation in (1)

$$\frac{G_{00}/6}{1/r_*^2} = \frac{T_{00}}{\rho_* c^2} \qquad (7)$$

by defining $G_{00} \equiv 6 n_r(1/r^2)$ and $T_{00} \equiv \rho_0 c^2$. That G_{00} is proportional to the n-ratio n_r demonstrates in a simple way that the Einstein equation is physically related to stresses in the PV.

The time varying luminosity of an AGN can be used to estimate the AGN's radius. A simplified calculation for a typical AGN [5, p.1110] leads to the radius $r_0 = 1.1 \times 10^{14}$cm. From (5) with $r = r_0$, this radius can be related to the AGN mass density ρ_0 via

$$\frac{\rho_0}{\rho_*} = n_0 \left(\frac{r_*}{r_0}\right)^2 \qquad (8)$$

where $n_0 = (m_0 c^2/r_0)/(m_* c^2/r_*)$. From previous investigations [2, 6], a reasonable n-ratio to assume for the AGN might be $n_0 = 0.5$, leading from (8) to

$$\frac{\rho_0}{\rho_*} = 0.5 \left(\frac{1.62 \times 10^{-33}}{1.1 \times 10^{14}}\right)^2 = 1.1 \times 10^{-94} \qquad (9)$$

for the relative mass density. Then the absolute density is

$$\begin{aligned} \rho_0 = 1.1 \times 10^{-94}\rho_* &= \\ &= 1.1 \times 10^{-94}\, 1.22 \times 10^{93} = 0.13\,[\text{gm/cm}^3] \end{aligned} \qquad (10)$$

which yields

$$m_0 = \frac{4\pi r_0^3 \rho_0}{3} =$$

$$= \frac{4\pi(1.1 \times 10^{14})^3 (0.13)}{3} = 7.2 \times 10^{41} [\text{gm}] \qquad (11)$$

for the mass of the AGN.

The standard calculation uses the black-hole/mass-accretion paradigm to determine the AGN mass and leads to the estimate $m_0 > 6.6 \times 10^{41}$ gm for the typical calculation referenced above. This result compares favorably with the 7.2×10^{41} gm estimate in (11) and yields the n-ratio $n_r = 0.44$.

Currently there is no generally accepted theory for the time variability in the luminosity of an AGN [1, 7]. As mentioned above, there is also no PV-acceptable line element to be used in the AGN modeling. As a substitute, the line elements for the generalized Schwarzschild solution of a point mass will be used to address the luminosity variability. Furthermore, because the differential geometry of the General theory is certainly not applicable for $r < r_* = 1.62 \times 10^{-33}$ [2], the point-mass solution will be treated as a model for a "hole" of radius r_* that leads from the visible universe into the PV.

If it assumed that the luminosity of the AGN is due to a large photon flux from the PV, through the "hole", and into the visible universe, then the corresponding luminosity will be proportional to the *coordinate* velocity of this flux. If it is further assumed that the flux excites material that has collected between the coordinate radii corresponding to the n-ratios $n_r = 0.5$ and $n_r = 1$, then both the variable luminosity and its uniformity at the surface of the AGN can be explained by the model, the uniformity resulting from the compact nature of the variable-flux source at the surface $r = r_*$. (The distortion of the PV by the collection of material between 0.5 and 1 is ignored in the rough model being pursued.)

The general solution [8, 9] to the Einstein field equations leading to the Schwarzschild line elements mentioned above is given in Appendix A. The magnitude of the relative coordinate velocity of a photon approaching or leaving the area of the point mass in a radial direction can be calculated from this solution as ($n = 1, 2, 3, \ldots$)

$$\beta_n(n_r) = \left| \frac{dr}{c\,dt} \right| = \left(\frac{g_{00}}{-g_{11}} \right)^{1/2} =$$

$$= (1 + 2^n n_r^n)^{(1-1/n)} \left(1 - \frac{2n_r}{(1 + 2^n n_r^n)^{1/n}} \right) \qquad (12)$$

whose plot as a function of n_r in Figure 1 shows β_n's behavior for $n = 3, 10, 20$. The vertical and horizontal axes run from 0 to 1. The approximate n-ratios for various astrophysical bodies are labeled on the $n = 3$ curve and include white dwarfs, neutron stars, and AGNs. The free Planck particle is labeled PP.

The existence of multiple solutions ($n = 1, 2, 3, \ldots$) in the spacetime geometry suggests a dynamic condition implying the possibility of a variable n or a composite solution "oscillating" between various values of n. For example, consider a solution oscillating between the $n = 10$ and $n = 20$ indices in the figure, where the relative flux velocities at $n_r = 0.5$ are 0.125 and 0.066 respectively. Since the luminosity is proportional to these flux velocities, the variation in luminosity changes by a factor of $0.125/0.066 \sim 2$ over the period of the oscillation. Again, as the source of the flux is the compact "hole" leading from the PV, the surface of the AGN is uniformly brightened by the subsequent flux scattered by the material intervening between the "hole" and the AGN surface at $n_r = 0.5$ where from (6)

$$r_0 = \frac{2m_0 c^2}{m_* c^2 / r_*} \qquad (13)$$

as $m = m_0$ at $r = r_0$.

"Earlier studies of galaxies and their central black holes in the nearby Universe revealed an intriguing linkage between the masses of the black holes and of the central 'bulges' of stars and gas in the galaxies. The ratio of the black hole and the bulge mass is nearly the same for a wide range of galactic sizes and ages. For central black holes from a million to many billions of times the mass of our sun, the black hole's mass is about one one-thousandth of the mass of the surrounding galactic bludge. ... This constant ratio indicates that the black hole and the bulge affect each others' growth in some sort of interactive relationship. ... The big question has been whether one grows before the other or if they grow together, maintaining their mass ratio throughout the entire process." [10] Recent measurements suggest that the constant ratio seen in nearby galaxies may not hold in the early more distant galaxies. The black holes in these young galaxies are much more massive compared to the bulges in the nearby galaxies, implying that the black holes started growing first.

The astrophysical measurements described in the preceding paragraph in terms of black holes could just as well be described by the PV model of the present paper, suggesting that the PV is the source of the energy and variability of the AGN and probably the primary gases (electrons and protons) of its galactic bulge.

Acknowledgment

Appendix A   Crothers point mass

The general solution [6,8,9] to the Einstein field equations for a point mass m at $r = 0$ consists of the infinite collection ($n = 1, 2, 3, \ldots$) of Schwarzschild-like equations with continuous, non-singular met-
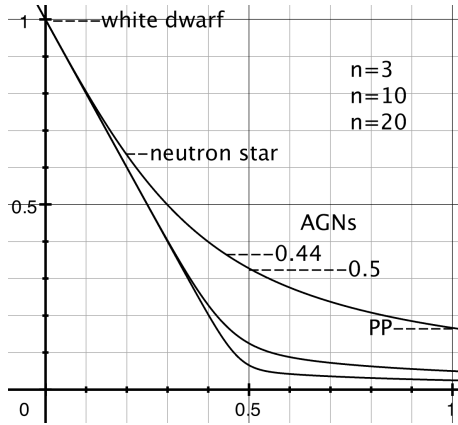
Fig. 1: The graph shows the relative coordinate velocity $\beta_n(n_r)$ plotted as a function of the n-ratio n_r for various indices n. Both axes run from 0 to 1. The approximate n-ratios corresponding to various astrophysical bodies are labeled on the $n=3$ curve and include white dwarfs ($n_r \sim 0.0002$), neutron stars (~ 0.2), and AGNs (the 0.44 value calculated from the black-hole model and the 0.5 assumed by the PV model). The free Planck particle is represented by PP (1). The intersections of the $n=10$ and $n=20$ curves with the 0.5 ordinate result in the relative velocities $\beta_{10}(0.5) = 0.125$ and $\beta_{20}(0.5) = 0.066$ respectively.

rics for $r > 0$:

$$ds^2 = g_{00}\, c^2 dt^2 + g_{11}\, dr^2 - R_n^2 \left(d\theta^2 + \sin^2\theta\, d\phi^2\right) \qquad (A1)$$

where

$$g_{00} = (1 - \alpha/R_n) \quad \text{and} \quad g_{11} = -\frac{(r/R_n)^{2n-2}}{g_{00}} \qquad (A2)$$

$$\alpha = \frac{2mc^2}{m_* c^2 / r_*} = 2rn_r \qquad (A3)$$

$$R_n = (r^n + \alpha^n)^{1/n} = r(1 + 2^n n_r^n)^{1/n} = \\ = \alpha(1 + 1/2^n n_r^n)^{1/n} \qquad (A4)$$

and

$$0 < n_r \left(= \frac{mc^2/r}{m_* c^2/r_*} \right) < 1 \qquad (A5)$$

where r is the *coordinate* radius from the point mass to the field point of interest, and m_* and r_* are the PP mass and Compton radius respectively.

The metrics in (A2) yield

$$g_{00} = 1 - \frac{2n_r}{(1 + 2^n n_r^n)^{1/n}} \longrightarrow 1 \qquad (A6)$$

$$-g_{11} = \frac{(1 + 2^n n_r^n)^{(2-2n)/n}}{g_{00}} \longrightarrow 1 \qquad (A7)$$

with

$$R_n = r(1 + 2^n n_r^n)^{1/n} \longrightarrow r \qquad (A8)$$

where the arrows lead to the far-field results for $n_r \to 0$. As expected, the n-ratio n_r in these equations is the sole variable that expresses the relative distortion of the PV due to the mass at $r = 0$.

Appendix B   Incompressible fluid

Outside a spherical mass of incompressible fluid (or any static mass of the same shape), the Schwarzschild line elements [3, 4] are the same as (A1) and (A2) except that for the fluid model

$$\alpha = \left(\frac{3}{\kappa \rho_0 c^2}\right)^{1/2} \sin^3 \chi_0 \qquad (B1)$$

where $\kappa = 8\pi G/c^4 = 6(1/r_*^2)/\rho_* c^2$ [2]

$$R_n = (r^n + \epsilon^n)^{1/n} \qquad (B2)$$

$$\sin \chi_0 = \left(\frac{\kappa \rho_0 c^2}{3}\right)^{1/2} (r_0^3 + \rho)^{1/3} \qquad (B3)$$

and $\epsilon = \epsilon(\rho_0, \chi_0)$ and $\rho = \rho(\rho_0, \chi_0)$ are constants, where ρ_0 represents the constant density of the fluid. The ratios in (B1) and (B3) can be expressed as

$$\frac{3}{\kappa \rho_0 c^2} = \frac{\rho_* r_*^2}{2\rho_0} \qquad (B4)$$

where $\rho_* (= m_*/(4\pi r_*^3/3))$ is the PP mass density.

Dividing (B1) by (B2) and using (B3) leads to

$$\frac{\alpha}{R_n} = \left(\frac{\kappa \rho_0 c^2}{3}\right) \frac{r_0^3 (1 + \rho/r_0^3)}{r(1 + \epsilon^n/r^n)^{1/n}}. \qquad (B5)$$

Inserting (B4) into (B5) then gives

$$\frac{\alpha}{R_n} = 2n_r \frac{1 + \rho/r_0^3}{(1 + \epsilon^n/r^n)^{1/n}} \qquad (B6)$$

after some manipulation, where the n-ratio

$$n_r = \frac{m_0 c^2/r}{m_* c^2/r_*}. \qquad (B7)$$

Here m_0 is defined in terms of the fluid density ρ_0 and the coordinate radius r_0, where $m_0 = (4\pi r_0^3/3)\rho_0$. The radius r_0 corresponds to the coordinate radius r_a in equation (32) of reference [4].

As pointed out in Appendix A, α/R_n (which is related to the PV distortion *exterior* to the mass) should be solely a function of the variable n_r as n_r is the only relative stress the static spherical mass as a whole can exert on the exterior vacuum. Consequently the variable r can only appear within the variable n_r. Therefor the denominator in (B6), and thus the incompressible-fluid model, are incompatible with the PV model.

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