The Planck Vacuum Physics Behind the Huygens Principle and the Propagator Theory for the Schrödinger Electron

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This paper reviews a small portion of the quantum-electrodynamic propagator model as viewed from the Planck vacuum (PV) theory. The nonrelativistic calculations suggest that the degenerate collection of Planck-particle cores (that pervade the invisible, negative-energy vacuum state) is responsible for the Huygens principle, the propagator theory, and the Feynman diagrams.

1 Introduction

The theoretical foundation [1] [2] [3] of the PV theory rests upon the unification of the Einstein, Newton, and Coulomb superforces:

$$\frac{c^4}{G}\left(=\frac{m_*c^2}{r_*}\right)=\frac{m_*^2G}{r_*^2}=\frac{e_*^2}{r_*^2} \tag{1}$$

where the ratio c^4/G is the curvature superforce that appears in the Einstein field equations. G is Newton's gravitational constant, c is the speed of light, m_* and r_* are the Planck mass and length respectively [4, p. 1234], and e_* is the massless bare charge. The fine structure constant is given by the ratio $\alpha = e^2/e_*^2$, where $(-e)$ is the observed electronic charge.

The two particle/PV coupling forces

$$F_c(r)=\frac{e_*^2}{r^2}-\frac{mc^2}{r} \quad \text{and} \quad F_*(r)=\frac{e_*^2}{r^2}-\frac{m_*c^2}{r} \tag{2}$$

the electron core $(-e_*, m)$ and the Planck-particle core $(-e_*, m_*)$ exert on the PV state, along with their coupling constants

$$F_c(r_c)=0 \quad \text{and} \quad F_*(r_*)=0 \tag{3}$$

and the resulting Compton radii

$$r_c=\frac{e_*^2}{mc^2} \quad \text{and} \quad r_*=\frac{e_*^2}{m_*c^2} \tag{4}$$

lead to the important string of Compton relations

$$r_cmc^2=r_*m_*c^2=e_*^2 \quad (=c\hbar) \tag{5}$$

for the electron and Planck-particle cores, where $\hbar$ is the reduced Planck constant. The electron and Planck-particle masses are m and m_* respectively. To reiterate, the equations in (2) represent the forces the free electron and Planck-particle cores exert on the PV space, a space that is itself pervaded by a degenerate collection of Planck-particle cores [5].

The Planck constant is a secondary constant whose structure can take different forms, e.g.

$$\hbar\,[\text{erg sec}]=r_cmc=r_*m_*c=\left(\frac{e_*^2}{r_*}\right)t_*=m_*c^2t_* \tag{6}$$

that are employed throughout the following text, where t_* $(=r_*/c)$ is the Planck time [4, p. 1234].

Furthermore, the energy and momentum operators expressed as

$$\widehat{E}=i\hbar\frac{\partial}{\partial t}=i(m_*c^2)t_*\frac{\partial}{\partial t}=i(m_*c^2)r_*\frac{\partial}{c\partial t} \tag{7}$$

and

$$c\,\widehat{\mathbf{p}}=-ic\hbar\nabla=-i(m_*c^2)r_*\nabla=-i(mc^2)r_c\nabla \tag{8}$$

will be used freely in what follows.

Section 2 re-examines the Schrödinger equation in light of the PV theory, the calculations concluding that the pervaded vacuum state is the source of the scattering in the propagator theory. Section 3 presents a nonrelativistic look at the Huygens principle and the propagator theory for the electron core.

2 Schrödinger equation

The inhomogeneous Schrödinger equation, where $H = H_0+V$ is the Hamiltonian operator, can be expressed as

$$\left(i\hbar\frac{\partial}{\partial t}-H\right)\psi(\mathbf{x},t)=0\,. \tag{9}$$

The free-space Hamiltonian is H_0 and V is some position and time-dependent potential that is assumed to slowly vanish in the remote past $(t\to-\infty)$ and in the remote future $(t\to+\infty)$. In free space $V = 0$ and (9) becomes

$$\left(i\hbar\frac{\partial}{\partial t}-H_0\right)\phi(\mathbf{x},t)=0\,. \tag{10}$$

For $t' > t$, the formal solution to (9) or (10) takes the form [6]

$$\psi(\mathbf{x},t')=T\exp\left[-i\int_t^{t'}dt''\,H(t'')/\hbar\right]\psi(\mathbf{x},t) \tag{11}$$

where T is the time-ordering operator whose details are unimportant here (see Appendix A). What is important is the decomposition of $\hbar$ $(=m_*c^2t_*)$ in the exponent of (11), leading to

$$\int_t^{t'}\frac{dt''\,H(t'')}{\hbar}=\int_t^{t'}\frac{dt''}{t_*}\frac{H(t'')}{m_*c^2}\,. \tag{12}$$

From the perspective of the PV theory, the normalization of dt'' by the Planck time t_* and H by the Planck-particle mass energy m_*c^2 strongly suggest that the scattering in the quantum-electrodynamic propagator theory is caused by the Planck-particle cores that pervade the vacuum state. This conclusion will be reinforced by the calculations to follow.

The normalized Hamiltonian operator H_0 can be expressed as

$$\begin{aligned} \frac{H_0}{m_*c^2} &= \frac{\text{“}p^2/2m\text{”}}{m_*c^2} = \frac{c\widehat{\mathbf{p}} \cdot c\widehat{\mathbf{p}}/2mc^2}{m_*c^2} \\ &= \frac{(-im_*c^2 r_* \nabla) \cdot (-imc^2 r_c \nabla)/2mc^2}{m_*c^2} = -\frac{r_c r_* \nabla^2}{2} \end{aligned} \tag{13}$$

where the equalities in (5) are used. Then the normalized Schrödinger equation becomes

$$ir_* \frac{\partial \phi}{c\partial t} - \frac{(-ir_c \nabla) \cdot (-ir_* \nabla)}{2} \phi = 0 \tag{14}$$

or

$$\left(it_* \frac{\partial}{\partial t} + \frac{r_c r_* \nabla^2}{2} \right) \phi = 0 \tag{15}$$

where t_* ($= r_*/c$) is the Planck time and the equations are dimensionless. The dimensionless aspect of the equations here and in what follows will help in recognizing the relationship between the Huygens principle and the propagator formalism.

The normalized inhomogeneous equation (9) becomes

$$\left(it_* \frac{\partial}{\partial t} + \frac{r_c r_* \nabla^2}{2} \right) \psi = \frac{V}{m_*c^2} \psi \tag{16}$$

where again the equation is dimensionless.

3 Electron-core propagator

Roughly speaking, the Huygens principle states that every point on a wavefront is itself the source of a spherical wavelet. In the present context, the Huygens principle takes the form [7, eqn. 6.29]

$$\phi(\mathbf{x}', t') = i \int d^3x \frac{G_0(\mathbf{x}', t'; \mathbf{x}, t)}{\hbar} \phi(\mathbf{x}, t) \quad \text{for} \quad t' > t$$

$$\phi(\mathbf{x}', t') = i \int d^3x \frac{G_0(\mathbf{x}', t'; \mathbf{x}, t)}{(m_*c^2)t_*} \phi(\mathbf{x}, t) \tag{17}$$

and

$$\psi(\mathbf{x}', t') = i \int d^3x \frac{G(\mathbf{x}', t'; \mathbf{x}, t)}{\hbar} \psi(\mathbf{x}, t) \quad \text{for} \quad t' > t$$

$$\psi(\mathbf{x}', t') = i \int d^3x \frac{G(\mathbf{x}', t'; \mathbf{x}, t)}{(m_*c^2)t_*} \psi(\mathbf{x}, t) \tag{18}$$

where the Green function propagators G_0 and G have the units ‘erg-sec per unit volume’. In the present paper, equations (17) and (18) are associated with what are defined as **internal-** and **external-**scattering processes respectively. The internal scattering refers to the free electron $\phi(\mathbf{x}, t)$ scattering off the pervaded PV space. The external scattering refers to the electron $\psi(\mathbf{x}, t)$ scattering off the pervaded PV space with an external potential $V(\mathbf{x}, t)$ perturbing that space. It will be seen in what follows that the units ‘erg-sec per unit volume’ almost define the ‘pervaded vacuum space’.

Now begins the calculation of the wave function ψ resulting from the continuous interaction of the free-electron wave function ϕ with the perturbed vacuum state. The calculation will not be carried to completion, but only far enough (equation (25)) to suggest that the wave scattering takes place between ϕ and the pervaded vacuum space. Furthermore, many of the details in the following calculations based on reference [7] are unimportant to the present needs; so the calculations are heavily referenced in case the reader is interested in those details.

For $t = \Delta t_1$ [7, eqn. 6.30]

$$\left(it_* \frac{\partial}{\partial t_1} + \frac{r_c r_* \nabla^2}{2} \right) \psi(\mathbf{x}_1, t_1) = \frac{V(\mathbf{x}_1, t_1)}{m_*c^2} \psi(\mathbf{x}_1, t_1) \tag{19}$$

and

$$\left(it_* \frac{\partial}{\partial t_1} + \frac{r_c r_* \nabla^2}{2} \right) \psi(\mathbf{x}_1, t_1) = 0 \tag{20}$$

for $t \neq \Delta t_1$. Equation (19) refers to an external scattering as defined above.

The new wave function due to the external perturbation V in (19) can be expressed as [7, eqn. 6.31]

$$\psi(\mathbf{x}_1, t_1) = \phi(\mathbf{x}_1, t_1) + \Delta\psi(\mathbf{x}_1, t_1) \tag{21}$$

so the Schrödinger equation yields (using (15) for ϕ)

$$\begin{aligned} &\left(it_* \frac{\partial}{\partial t_1} + \frac{r_* r_c \nabla^2}{2} \right) \Delta\psi(\mathbf{x}_1, t_1) \\ &= \frac{V(\mathbf{x}_1, t_1)}{m_*c^2} \left[\phi(\mathbf{x}_1, t_1) + \Delta\psi(\mathbf{x}_1, t_1) \right] . \end{aligned} \tag{22}$$

It can be shown that the second terms on the left and right sides of (22) can be dropped [7, eqn.6.35], leading to

$$it_* \frac{\partial}{\partial t_1} \Delta\psi(\mathbf{x}_1, t_1) = \frac{V(\mathbf{x}_1, t_1)}{m_*c^2} \phi(\mathbf{x}_1, t_1) \tag{23}$$

which to first order in Δt_1 yields

$$\Delta\psi(\mathbf{x}_1, t_1 + \Delta t_1) = -i \frac{V(\mathbf{x}_1, t_1)}{m_*c^2} \phi(\mathbf{x}_1, t_1) \frac{\Delta t_1}{t_*} \tag{24}$$

where the differential $\Delta\psi(\mathbf{x}_1, t_1)$ coming from the approximation is ignored compared to the $\phi(\mathbf{x}_1, t_1)$ on the right side of (24).

For two consecrative time periods $\Delta t_1 \Delta t_2$, with an infinite past [where $\psi(x') \rightarrow \phi(x')$], it can be argued that [7, eqn. 6.43]

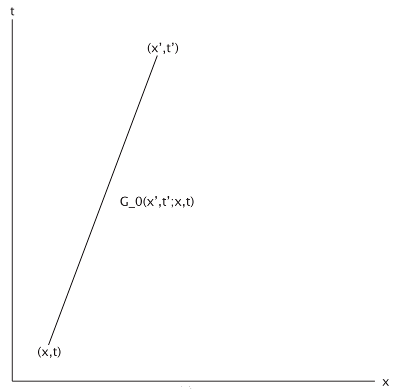

Fig. 1: The Feynman diagram for the propagation of the electron core $(-e_*, m)$ from (x, t) to (x', t') with no external scattering.

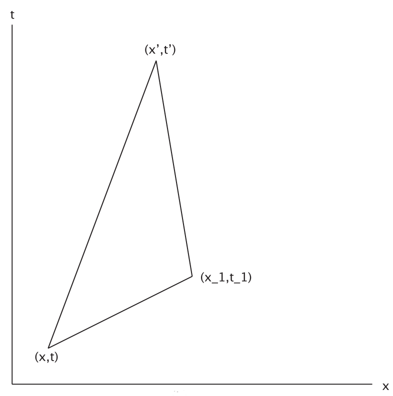

Fig. 2: The Feynman diagram for the propagation of the electron core $(-e_*, m)$ from (x, t) to (x', t') with one external scattering at (x_1, t_1).

$$
\begin{aligned}
\psi(x') = \quad & \phi(x') + \int d^3x_1 \frac{\Delta t_1}{t_*} \, G_0(x'; 1) \frac{V(1)}{m_* c^2} \phi(1) \\
& + \int d^3x_1 \frac{\Delta t_2}{t_*} \, G_0(x'; 2) \frac{V(2)}{m_* c^2} \phi(2) \\
& + \int d^3x_1 \frac{\Delta t_1}{t_*} d^3x_2 \frac{\Delta t_2}{t_*} \, G_0(x'; 2) \\
& \frac{V(2)}{m_* c^2} \, G_0(x'; 1) \, \frac{V(1)}{m_* c^2} \phi(1)
\end{aligned}
\tag{25}
$$

where the obvious notations $(x) \equiv (\mathbf{x}, t)$ and $\phi(2) \equiv \phi(x_2)$ are used. The four terms in (25) represent respectively the propagation from (x, t) to (x', t'): a) as a free particle with no external scatterings; b) with one scattering at (x_1, t_1); c) with one scattering at (x_2, t_2); and d) with a double scattering at (x_1, t_1) and (x_2, t_2) in succession. The representations of these scatterings in Figures 1-4 are called Feynman diagrams [7, eqn. 6.43], where the horizontal axis represents space and the vertical axis represents time.

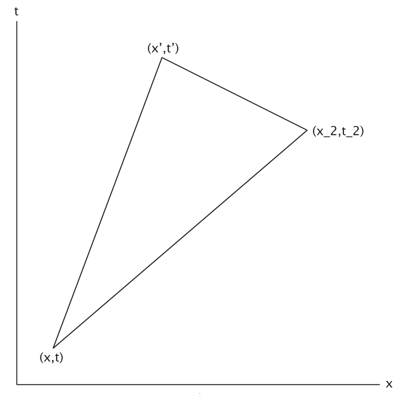

Fig. 3: The Feynman diagram for the propagation of the electron core $(-e_*, m)$ from (x, t) to (x', t') with one external scattering at (x_2, t_2).

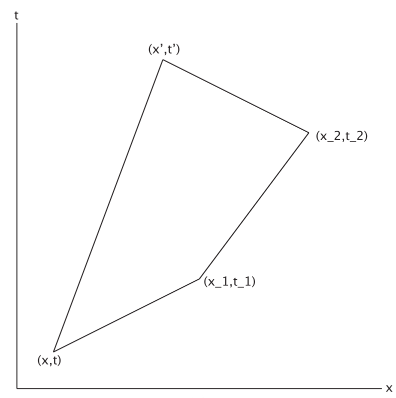

Fig. 4: The Feynman diagram for the propagation of the electron core $(-e_*, m)$ from (x, t) to (x', t') with a double external scattering at (x_1, t_1) and (x_2, t_2).

4 Conclusions and comments

A close examination of the previous calculations strongly suggests that the PV theory, which envisions a vacuum space pervaded by a degenerate collection of Planck-particle cores, provides a fundamental explanation for the Huygens principle and the scattering associated with the quantum-electrodynamic propagator formalism.

The retarded Green function G_0^+ associated with the Green function $G_0(\mathbf{x}', t'; \mathbf{x}, t)$ in equation (17) and in Figure 1 is given by the equations [7, eqn. 6.60]

$$
\left(i\hbar \frac{\partial}{\partial t'} - H_0(x') \right) \frac{G_0^+(x'; x)}{\hbar} = \delta^3(\mathbf{x}' - \mathbf{x}) \delta(t' - t) \tag{26}
$$

for $t' > t$ and $G_0^+(x'; x) = 0$ for $t' < t$, where $x' = (\mathbf{x}', t')$ and $x = (\mathbf{x}, t)$; or

$$
\left(i t_* \frac{\partial}{\partial t'} + r_c r_* \nabla_{x'}^2 \right) G_0^+(x'; x) = \delta^3(\mathbf{x}' - \mathbf{x}) [t_* \delta(t' - t)] \tag{27}
$$

where the parenthesis on the left and the bracket on the right of (27) are dimensionless.

Appendix A: Time-ordering operator T

The time-ordering operator [6] is defined by

$$T \exp\left[-i\int_{t}^{t'} dt'' H(t'')/\hbar\right] \equiv \tag{A1}$$

$$\sum_{n=0}^{\infty} \frac{1}{n!}\left(\frac{-i}{\hbar}\right)^n \int_{t}^{t'} dt_1 \ldots \int_{t}^{t_{n-1}} dt_n H(t_1)\ldots H(t_n) \tag{A2}$$

$$= \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int_{t}^{t'} \frac{dt_1}{t_*} \ldots \int_{t}^{t_{n-1}} \frac{dt_n}{t_*} \frac{H(t_1)}{m_* c^2} \ldots \frac{H(t_n)}{m_* c^2} \tag{A3}$$

where the final equality comes from the decomposition of the Planck constant, $\hbar = m_* c^2 t_*$, in (A2).

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