Limits to the Validity of the Einstein Field Equations and General Relativity from the Viewpoint of the Negative-Energy Planck Vacuum State

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It is assumed in what follows that the negative-energy Planck vacuum (see the appendix) is the underlying "space" upon which the spacetime equations of General Relativity operate. That is, General Relativity deals with the spacetime aspects of the Planck vacuum (PV). Thus, as the PV appears continuous only down to a certain length ($l = 5r_*$ or greater, perhaps), there is a limit to which the differential geometry of the general theory is valid, that point being where the "graininess" ($l \sim r_* > 0$) of the vacuum state begins to dominate. This aspect of the continuity problem is obvious; what the following deals with is a demonstration that the Einstein equation is tied to the PV, and that the Schwarzschild line elements derived from this equation may be significantly limited by the nature of that vacuum state.

A spherical object of mass m and radius r exerts a relative curvature force

$$n_r = \frac{mc^2/r}{m_* c^2/r_*} \tag{1}$$

on the negative-energy PV and the spacetime of General Relativity, where m_* and r_* are the Planck particle (PP) mass and Compton radius respectively. For example: a white dwarf of mass $9{\times}10^{32}$ gm and radius $3{\times}10^{8}$ cm exerts a curvature force equal to $2.7{\times}10^{45}$ dyne; while a neutron star of mass $3{\times}10^{33}$ gm and radius $1{\times}10^{6}$ cm exerts a force of $2.7{\times}10^{48}$ dyne. Dividing these forces by the $1.21{\times}10^{49}$ dyne force in the denominator leads to the n-ratios $n_r = 0.0002$ and $n_r = 0.2$ at the surface of the white dwarf and neutron star respectively. As the free PP curvature force $m_* c^2/r_*$ is assumed to be the maximum such force that can be exerted on spacetime and the PV, the n-ratio is limited to the range $n_r < 1$.

The numerator in the first of the following two expressions for the Einstein field equation derived in the appendix

$$G_{\mu\nu} = \frac{8\pi T_{\mu\nu}}{m_* c^2/r_*} \quad \text{and} \quad \frac{G_{\mu\nu}/6}{1/r_*^2} = \frac{T_{\mu\nu}}{\rho_* c^2} \tag{2}$$

is normalized by this maximum curvature force. The second expression ties the Einstein equation to the PPs making up the degenerate PV, where $1/r_*^2$ and $\rho_* c^2$ are the PPs' Gaussian curvature and mass-energy density respectively. The denominators in the second expression represent the Planck limits for the maximum curvature and the maximum equivalent mass-energy density respectively, both limits corresponding to $n_r = 1$. For larger n_r, the equations of General Relativity, derived for a continuum using differential geometry, break down for the reasons already cited.

The limits on the Einstein equation carry over, of course, to results derived therefrom. A simple example is the case of Schwarzschild's point-mass derivation [1]. Its more general form [2] for a point mass m at $r = 0$ consists of the infinite collection ($n = 1, 2, 3, \cdots$) of Schwarzschild-like equations with continuous, non-singular metrics for $r > 0$:

$$ds^2 = \left(1 - \frac{\alpha}{R_n}\right) c^2 dt^2 - \frac{(r/R_n)^{2n-2}\, dr^2}{1 - \alpha/R_n} - R_n^2\, (d\theta^2 + \sin^2\theta\, d\phi^2), \tag{3}$$

where

$$\alpha = \frac{2mc^2}{c^4/G} = 2\,\frac{mc^2}{m_* c^2/r_*} \tag{4}$$

and

$$R_n = (r^n + \alpha^n)^{1/n} = r(1 + 2^n n_r^n)^{1/n} = \alpha(1 + 1/2^n n_r^n)^{1/n}, \tag{5}$$

where n_r is given by (1) with r in this case being the *coordinate* radius from the point mass to the field point of interest. The original Schwarzschild solution [1] corresponds to $n = 3$. Here again, r is restricted to the range $r > r_*$ due to the previous continuity arguments leading to $n_r < 1$.

The plots of the time metric

$$g_{00} = g_{00}(n; n_r) = 1 - \frac{\alpha}{R_n} = 1 - \frac{2n_r}{(1 + 2^n n_r^n)^{1/n}} \tag{6}$$

as a function of n_r in Figure 1 show its behavior as n increases from 1 to 20. The vertical axis represents g_{00} from 0 to 1 and the horizontal axis n_r over the same range. The limiting case as n increases without limit yields

$$g_{00} = 1 - 2n_r \tag{7}$$

for $n_r \leqslant 0.5$. The same limit leads from (3) to the line element

$$ds^2 = (1 - 2n_r)\, c^2 dt^2 - \frac{dr^2}{(1 - 2n_r)} - r^2\, (d\theta^2 + \sin^2\theta\, d\phi^2), \tag{8}$$

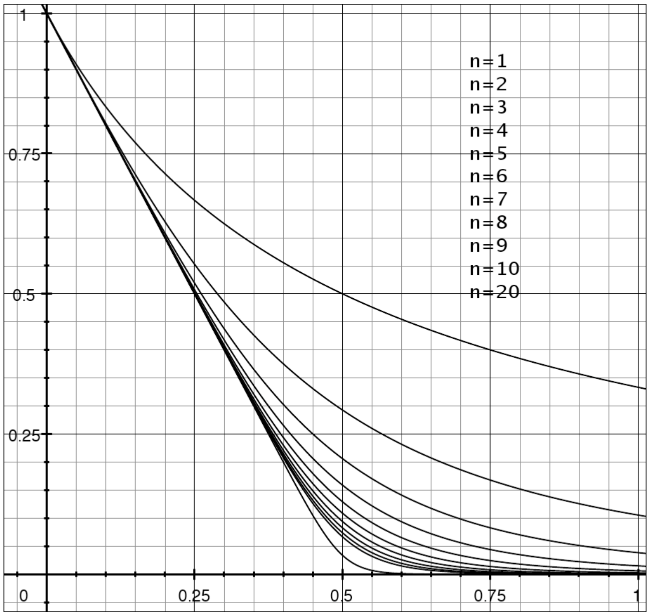

Fig. 1: The graph shows the time metric $g_{00} = g_{00}(n; n_r)$ plotted as a function of the n-ratio n_r for various indices n. Both axes run from 0 to 1. The "dog-leg" in the curves approaches the point (0.5,0) from above ($n_r > 0.5$) as n increases, the limiting case $n \to \infty$ yielding the metric $g_{00} = 1 - 2n_r$ for $n_r \leqslant 0.5$.

for $n_r \leqslant 0.5$. This is the same equation as the standard black-hole/event-horizon line element [3, p.360] except for the reduced range in n_r. Mathematically, the metrics in (3) are non-singular down to any $r > 0$, but we have already seen that this latter inequality should be replaced by $r > r_* > 0$ as $n_r < 1$.

As n_r increases from 0.5, it is assumed that a point is reached prior to $n_r = 1$ where the curvature stress on the PV is sufficient to allow energy to be released from the PV directly into the visible universe. A related viewpoint can be found in a closely similar, field-theoretic context:

> "[This release of energy] is in agreement with observational astrophysics, which in respect of high-energy activity is all of explosive outbursts, as seen in the QSOs, the active galactic nuclei, etc. The profusion of sites where X-ray and γ-ray activity is occurring are in the present [quasi-steady-state] theory sites where the creation of matter is currently taking place" [4, p. 340].

In summary: the obvious restraint on the Einstein field equations is that their time and space differentials be an order of magnitude or so greater than r_*/c and r_* respectively; and that $n_r < 1$, with some thought being given to the application of the equations in the region where $0.5 < n_r < 1$.

Appendix The Planck vacuum

The PV [5] is a uni-polar, omnipresent, degenerate gas of negative-energy PPs which are characterized by the triad (e_*, m_*, r_*), where e_*, m_*, and r_* $(\lambda_*/2\pi)$ are the PP charge, mass, and Compton radius respectively. The vacuum is held together by van der Waals forces. The charge e_* is the bare (true) electronic charge common to all charged elementary particles and is related to the observed electronic charge e through the fine structure constant $\alpha = e^2/e_*^2$ which is a manifestation of the PV polarizability. The PP mass and Compton radius are equal to the Planck mass and length respectively. The particle-PV interaction is the source of the gravitational ($G = e_*^2/m_*^2$) and Planck ($\hbar = e_*^2/c$) constants, and the string of Compton relations

$$r_* m_* = \cdots = r_c m = \cdots = e_*^2/c^2 = \hbar/c \tag{A1}$$

relating the PV and its PPs to the observed elementary particles, where the charged elementary particles are characterized by the triad (e_*, m, r_c), m and r_c being the mass and Compton radius $(\lambda_c/2\pi)$ of the particle (particle spin is not yet included in the theory). The zero-point random motion of the PP charges e_* about their equilibrium positions within the PV, and the PV dynamics, are the source of the quantum vacuum [6] [7]. Neutrinos appear to be phonon packets that exist and propagate within the PV [8].

The Compton relations (A1) follow from the fact that an elementary particle exerts two perturbing forces on the PV, a curvature force mc^2/r and a polarization force e_*^2/r^2:

$$\frac{mc^2}{r} = \frac{e_*^2}{r^2} \qquad \Longrightarrow \qquad r_c = \frac{e_*^2}{mc^2} \tag{A2}$$

whose magnitudes are equal at the particle's Compton radius r_c.

Equating the first and third expressions in (A1) leads to $r_* m_* = e_*^2/c^2$. Changing this result from Gaussian to MKS units yields the free-space permittivities

$$\epsilon_0 = \frac{1}{\mu_0 c^2} = \frac{e_*^2}{4\pi r_* m_* c^2} \quad \text{[mks]}, \tag{A3}$$

where $\mu_0/4\pi = r_* m_*/e_*^2 = r_c m/e_*^2 = 10^{-7}$ in MKS units. Converting (A3) back into Gaussian units gives

$$\epsilon = \frac{1}{\mu} = \frac{e_*^2}{r_* m_* c^2} = 1 \tag{A4}$$

for the permittivities.

A feedback mechanism in the particle-PV interaction leads to the Maxwell equations and the Lorentz transformation [5] [9].

General Relativity describes the spacetime-curvature aspects of the PV. The ultimate curvature force

$$\frac{c^4}{G} = \frac{m_* c^2}{r_*} \tag{A5}$$

that can be exerted on spacetime and the PV is due to a free PP. An astrophysical object of mass m exerts a curvature force equal to mc^2/r at a coordinate distance r from the center of the mass. Equation (A5) leads to the ratio

$$\frac{c^4}{8\pi G} = \frac{1}{6}\,\frac{\rho_* c^2}{1/r_*^2}\,, \tag{A6}$$

where $\rho_* \equiv m_*/(4\pi r_*^3/3)$ is the PP mass density and $1/r_*^2$ is its Gaussian curvature. The Einstein equation including the cosmological constant Λ can then be expressed as

$$\frac{(G_{\mu\nu} + \Lambda g_{\mu\nu})/6}{1/r_*^2} = \frac{T_{\mu\nu}}{\rho_* c^2} \tag{A7}$$

tying the differential geometry of Einstein to the PPs in the negative-energy PV. In this form both sides of the equation are dimensionless.

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