The Relativistic Energy-Momentum Equation in the Planck Vacuum Theory

William C. Daywitt (European Journal of Applied Physics for December 2023)

***Abstract*—This paper examines the relativistic energy-momentum equation and its use in the photon-electron Compton scattering calculations. It provides a better understanding of that equation and reveals the reason for particle spin.**

***Index Terms*—Energy-Momentum Equation, Compton Scattering, Spin Energy, Planck Vacuum Theory.**

I. Introduction

THE theoretical foundation [1]—[5] of the PV theory rests upon the unification of the Einstein, Newton, and Coulomb superforces:

$$\frac{c^4}{G_d}\left(=\frac{m_* c^2}{r_*}\right)=\frac{m_*^2 G}{r_*^2}=\frac{e_*^2}{r_*^2}\rightarrow r_* m_* c=\frac{e_*^2}{c}\quad (\hbar) \tag{1}$$

where the ratio c^4/G_d is the curvature superforce that appears in the Einstein field equations. G is Newton's gravitational constant, c is the speed of light, m_* and r_* are the Planck mass and length respectively [6, p.1234], and e_* is the massless bare (or coupling) charge. The Planck time is $t_* = r_*/c$ [6, p.1233]. The fine structure constant is given by the ratio $\alpha \equiv e^2/e_*^2$, where e is the observed electronic charge magnitude. The ratio e_*^2/c to the right of the arrow is the spin coefficient for the Planck particle (PP), the proton, and the electron cores, where $\hbar$ is the reduced Planck constant. One of the e_*s in e_*^2 belongs to the PP under consideration and the other to any one of the remaining PPs making up the PV state.

The electron, proton, and PP Dirac cores associated with the PV theory defined above are

$$(\pm e_*, m_e)\quad (\pm e_*, m_p)\quad \text{and}\quad (\pm e_*, m_*) \tag{2}$$

respectively. The $\pm$ signs in the equations include the antiparticles. Their coupling to the highly energetic PV state is through the spin equations

$$r_e m_e c = r_p m_p c = \frac{e_*^2}{c} = r_* m_* c \tag{3}$$

where it appears that *particle spin and (3) exist to separate (in magnitude) the masses on the left from the mass m_* on the right*. The spin is generated in the zero-point PV oscillations [7].

All of the preceding equations are fixed in the sense that their structure is determined at the high PV energy level. This level is roughly nineteen (proton cores) to twenty-two (electron cores) orders-of-magnitude more energetic than the processes taking place at the electron or proton levels. The masses m_e and m_p are assumed to be created along with their Compton radii r_e and r_p in (3) within the PV state.

From equation (1) the gravity bodies in the PV theory are described by the gravitational constants [8]

$$G_d = \frac{r_* c^2}{m_*} = \frac{e_*^2}{m_*^2} = G \tag{4}$$

where it is noted that G_d *does not react to photons because it contains no charge e_*.*

II. Conclusions and Comments

The total (charge and mass) electron core coupling force acting on the PV state in equation (1) is [9]

$$F_e(r) = \frac{e_*^2}{r^2} - \frac{m_e m_* G}{r r_*} = \frac{e_*^2}{r^2} - \frac{m_e c^2}{r} \tag{5}$$

with the result that the core propagates through free space unattenuated because its coupling force

$$F_e(r_e) = 0 \tag{6}$$

vanishes at $r = r_e$. From Appendix A, r_e *is a constant of the motion that completes equation (6).*

The covariant Dirac equation (A1) shows that the Compton radius r_e is a constant of the motion in the spin equations. Therefor it must be possible to create an equation that includes r_e automatically without being seen. The following relativistic energy-momentum equation is just such an equation

$$W^2 = c^2 P^2 + m_e^2 c^4 = c^2 P^2 + c^2 (m_e c)^2 \tag{7}$$

where W is the total energy, cP is the total kinetic energy, and P is the relativistic momentum. Since the r_e in (7) includes particle-spin, W and P must also include particle-spin. In that case, the solution to (7) can be expressed as

$$W/m_e c^2 = \pm\left[1 + (P/m_e c)^2\right]^{1/2} \tag{8}$$

which includes an antiparticle.

Assume that a quantum of energy $h\nu$ with a momentum $h\nu/c$ moves along the positive x-axis in outer space and strikes a free electron core at rest; and then departs at an angle θ to the axis, with a wavelength increase $\Delta\lambda$. Then [10] [11]

$$\Delta\lambda = \frac{h}{m_e c}(1-\cos\theta) = 2\pi r_e (1-\cos\theta) \tag{9}$$

where h is the Planck constant and

$$r_e = \frac{e_*^2/c}{m_e c}. \tag{10}$$

It is also shown that

$$\frac{\Delta\lambda}{\lambda} = \frac{cP}{h\nu + m_e c^2}\cos\psi = \frac{P}{h\nu/c + m_e c}\cos\psi \tag{11}$$

where λ is the original wavelength. The energy-momentum equation is essential to the above Compton scattering calculations.

The previous calculations lead immediately to the proton core by replacing r_e with r_p.

Appendix A
Constant of the Motion

The covariant form [12, p.90] [Appendix B] of the Dirac equation for the electron and positron cores *combined* can be expressed as:

$$\left(i\hbar\gamma^\mu \frac{\partial}{\partial x^\mu} - m_e c\right)\psi = \left(i\frac{e_*^2}{c}\gamma^\mu \frac{\partial}{\partial x^\mu} - m_e c\right)\psi \quad \text{(A1)}$$

$$= i\frac{e_*^2}{c}\left[\gamma^0 \frac{\partial}{\partial x^0} + \begin{pmatrix} 0 & \sigma_j \\ -\sigma_j & 0 \end{pmatrix} \frac{\partial}{\partial x^j}\right]\psi - m_e c\psi = 0 \quad \text{(A2)}$$

where dividing through by $m_e c$ and recombining leads to

$$ir_e\left[\gamma^\mu \frac{\partial}{\partial x^\mu}\right]\psi - \psi = 0 \quad \text{with} \quad r_e = \frac{e_*^2/c}{m_e c} \quad \text{(A3)}$$

and where the j partial-derivative terms in (A2) are summed over $j = 1, 2, 3$. The character of equation (A2) implies that the equations here include particle spin.

It is noted from (A3) that r_e *is a constant of the motion as far as the wavefunction* ψ *is concerned*.

Appendix B
The γ And β Matrices

The 4x4 γ, β, and α_i matrices used in the Dirac theory are defined here: where [12, p.91]

$$\gamma^0 \equiv \beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \quad \text{(B1)}$$

and ($j = 1, 2, 3$)

$$\gamma^j \equiv \beta\alpha_j = \begin{pmatrix} 0 & \sigma_j \\ -\sigma_j & 0 \end{pmatrix} \quad \text{(B2)}$$

and where I is the 2x2 unit matrix and

$$\alpha_j = \begin{pmatrix} 0 & \sigma_j \\ \sigma_j & 0 \end{pmatrix} \quad \text{(B3)}$$

where the σ_j are the 2x2 Pauli spin matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \text{(B4)}$$

and $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, \alpha_3)$. The zeros in (B1)–(B3) are 2x2 null matrices.

The zeros on the right end of (A2) and (A3) represent 4x4 null matrices.

The coordinates x^μ are

$$x^\mu = (x^0, x^1, x^2, x^3) \quad \text{(B5)}$$

where $x^0 \equiv ct$.

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