Spin in the Dirac and Schrödinger Equations According to the Planck Vacuum Theory

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***Abstract*—This paper examines the concept of particle spin as it applies to the Dirac and Schrödinger equations in the Planck vacuum (PV) theory. Results show: that the reduced Planck constant is equal to the spin coefficient for the Planck particle (PP), the proton, and the electron cores; that the well-known energy and momentum operators can be used to derive the Schrödinger equation; that the wavefunctions for both equations are related to the electron Compton radius; and that the relativistic spin operator is derived from the Dirac equation.**

The spin and mass of the electron or proton cores are created simultaneously within the PV and transmitted into free space.

***Index Terms*—Particle Spin, Planck Constant, Planck Vacuum Theory.**

I. Introduction

THE theoretical foundation [1] [2] [3] [4] of the PV theory rests upon the unification of the Einstein, Newton, and Coulomb superforces:

$$\frac{c^4}{G_d}\left(=\frac{m_*c^2}{r_*}\right)=\frac{m_*^2G}{r_*^2}=\frac{e_*^2}{r_*^2}\to r_*m_*c=\frac{e_*^2}{c}\quad(\hbar)\tag{1}$$

where the ratio c^4/G_d is the curvature superforce that appears in the Einstein field equations. G is Newton's gravitational constant, c is the speed of light, m_* and r_* are the Planck mass and length respectively [5, p.1234], and e_* is the massless bare (or coupling) charge. The Planck time is $t_*=r_*/c$ [5, p.1233]. The fine structure constant is given by the ratio $\alpha\equiv e^2/e_*^2$, where e is the observed electronic charge magnitude. The ratio e_*^2/c to the right of the arrow is the spin coefficient for the PP, the proton, and the electron cores, where $\hbar$ is the reduced Planck constant. One of the e_*s in e_*^2 belongs to the PP under consideration and the other to any one of the remaining PPs making up the PV state.

The electron, proton, and PP Dirac cores associated with the PV theory defined above are

$$(\pm e_*,m_e)\quad(\pm e_*,m_p)\quad\text{and}\quad(\pm e_*,m_*)\tag{2}$$

respectively. The $\pm$ signs in the equations include the antiparticles. Their coupling to the highly energetic PV state is through the spin equations

$$r_em_ec=r_pm_pc=\frac{e_*^2}{c}=r_*m_*c.\tag{3}$$

The spin is generated in the zero-point PV oscillations [6].

All of the preceding equations are fixed in the sense that their structure is determined at the high PV energy level. This level is roughly nineteen (proton cores) to twenty-two (electron cores) orders-of-magnitude more energetic than the processes taking place at the electron or proton levels. The masses m_e and m_p are assumed to be created along with their Compton radii r_e and r_p in (3) within the PV state.

II. Planck Vacuum State

The PV state consists of a continuum that is pervaded by a degenerate collection of PP cores. Since the separate PPs in that state are roughly confined to a sphere of radius r_*, they are nonrelativistic [6]. The energy and momentum operators operate on this state to produce the electron and proton cores in free space.

III. Energy and Momentum Operators

In the PV theory the well-known energy and momentum operators [7, p.20] take the form

$$\widehat{E}=i\hbar\frac{\partial}{\partial t}=i\frac{e_*^2}{c}\frac{\partial}{\partial t}\tag{4}$$

and

$$\widehat{p}=-i\hbar\nabla=-i\frac{e_*^2}{c}\nabla\quad\text{and}\quad\widehat{p}^\dagger=+i\frac{e_*^2}{c}\nabla\tag{5}$$

where $\widehat{p}^\dagger$ is the complex-conjugate operator to $\widehat{p}$. The spin coefficient e_*^2/c couples the electron and proton cores in (2) to the PV state in (1).

The particle spin $\overrightarrow{S}$ is

$$\overrightarrow{S}=\frac{e_*^2}{c}\overrightarrow{\sigma}\quad\text{with}\quad r_*=\frac{e_*^2/c}{m_*c}\tag{6}$$

where r_* is the PP Compton radius [8] [9]. Equation (3) gives the electron- and proton-core derivatives of r_*.

IV. Dirac Equation

The covariant form [10, p.90] [Appendix A] of the Dirac equation for the electron and positron cores *combined* can be expressed as:

$$\left(i\hbar\gamma^\mu\frac{\partial}{\partial x^\mu}-m_ec\right)\psi=\left(i\frac{e_*^2}{c}\gamma^\mu\frac{\partial}{\partial x^\mu}-m_ec\right)\psi\tag{7}$$

$$=i\frac{e_*^2}{c}\left[\gamma^0\frac{\partial}{\partial x^0}+\begin{pmatrix}0&\sigma_j\\-\sigma_j&0\end{pmatrix}\frac{\partial}{\partial x^j}\right]\psi-m_ec\psi=0\tag{8}$$

where dividing through by m_ec and recombining leads to

$$ir_e\left[\gamma^\mu\frac{\partial}{\partial x^\mu}\right]\psi-\psi=0\quad\text{with}\quad r_e=\frac{e_*^2/c}{m_ec}\tag{9}$$

where the j partial-derivative terms in (8) are summed over $j=1,2,3$.

V. Nonrelativistic Schrödinger Equation

Using (4) and (5), the Schrödinger equation [7, p.20] with the wavefunction ϕ can be expressed as

$$\left(\widehat{E} - H\right)\phi = \left(ie_*^2 \frac{\partial}{c\partial t} - H\right)\phi \tag{10}$$

$$= \left(ie_*^2 \frac{\partial}{c\partial t} - \frac{\widehat{p}^2}{2m_e}\right)\phi = \left(ie_*^2 \frac{\partial}{c\partial t} - \frac{e_*^4 \nabla^2}{2m_e c^2}\right)\phi \tag{11}$$

$$= \frac{e_*^2}{c}\left(i\frac{\partial}{\partial t} - \frac{e_*^2 \nabla^2}{2m_e c}\right)\phi = \frac{e_*^2}{c}\left(i\frac{\partial}{\partial t} - \frac{r_e^2 \nabla^2}{2t_e}\right)\phi = 0 \tag{12}$$

where the 2 comes from the fact that the energy of the PPs within the PV state are nonrelativistic. The electron time constant $t_e (= r_e/c)$ yields the distance r_e a photon travels in t_e seconds. The $\widehat{p}^2$ is the complex conjugate square to $\widehat{p}$.

From (12) the Schrödinger equation is

$$\frac{e_*^2}{c}\left(i\frac{\partial}{\partial t} - \frac{r_e^2 \nabla^2}{2t_e}\right)\phi = 0. \tag{13}$$

Normalizing (13) by $m_e c$ gives

$$r_e\left(i\frac{\partial}{\partial t} - \frac{r_e^2 \nabla^2}{2t_e}\right)\phi = 0 \tag{14}$$

and the H operator yields

$$\frac{H}{m_e c} = r_e\left(\frac{r_e^2 \nabla^2}{2t_e}\right). \tag{15}$$

VI. Conclusions and Comments

The energy and momentum operators (4) and (5) operate on the PV state (1) which leads to the first two spins in (3) given by the spin coefficient. The PV system communicates this spin to the electron and proton cores by the equations in (3). Although the electron and proton cores are orders-of-magnitude less energetic than the PP core, the $r \cdot m$ products in (3) provide the spin&mass transfer from the PV to the first two cores. It is impotent to notice that, *by the very definition of the word 'covariance', the covariance of the Dirac equation depends upon the covariance of the particle spins in (3).*

The PV theory modifies the original Dirac equation by replacing the $\hbar$ in the left side of equation (7) by the spin coefficient e_*^2/c in the right side. The resulting change is shown in equation (8), where now the spin coefficient appears on the far left and momentum appears on the right. Dividing through by the momentum $m_e c$ then leads to the dimensionless equation (9), where the electron Compton radius [8] [9] normalizes the partial derivatives.

The Dirac and Schrödinger equations compared are

$$ir_e\left[\gamma^\mu \frac{\partial}{\partial x^\mu}\right]\psi - \psi = 0 \quad \text{with} \quad r_e = \frac{e_*^2/c}{m_e c} \tag{16}$$

and

$$r_e\left(i\frac{\partial}{\partial t} - \frac{r_e^2 \nabla^2}{2t_e}\right)\phi = 0 \tag{17}$$

where *the coefficient in (16) is the relativistic spin operator.*

Finally, to be clear the PV state is not a vacuum state. It is a PP-perturbed continuum that is so many orders-of-magnitude more energetic than the free-space particle cores that those cores do not perturb the PV state.

The previous calculations lead immediately to the proton core by replacing r_e with r_p.

Appendix A
The γ And β Matrices

The 4x4 γ, β, and α_i matrices used in the Dirac theory are defined here: where [10, p.91]

$$\gamma^0 \equiv \beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \tag{A1}$$

and $(j = 1, 2, 3)$

$$\gamma^j \equiv \beta\alpha_j = \begin{pmatrix} 0 & \sigma_j \\ -\sigma_j & 0 \end{pmatrix} \tag{A2}$$

and where I is the 2x2 unit matrix and

$$\alpha_j = \begin{pmatrix} 0 & \sigma_j \\ \sigma_j & 0 \end{pmatrix} \tag{A3}$$

where the σ_j are the 2x2 Pauli spin matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{A4}$$

and $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, \alpha_3)$. The zeros in (A1)–(A3) are 2x2 null matrices.

The zeros on the right end of (8) and (9) represent 4x4 null matrices.

The coordinates x^μ are

$$x^\mu = (x^0, x^1, x^2, x^3) \tag{A5}$$

where $x^0 \equiv ct$.

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