

# The Schrödinger Equation and the Time-Ordering Operator T of the Quantum Field Theory as Viewed in the Planck Vacuum Theory

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**Abstract**—This paper derives the Schrödinger equation and examines the corresponding time-ordering operator T of the quantum field theory. Results show that the equation supports a particle spin while the quantum field does not. This difference is to be expected as the quantum field result describes a field rather than a particle core.

It appears that both the spin and the mass of the particle are created in the zero-point Planck vacuum (PV) oscillations of the PV state.

**Index Terms**—Schrödinger Equation, Time-Ordering Operator T, Planck Vacuum Theory.

## I. INTRODUCTION

THE theoretical foundation [1] [2] [3] [4] of the PV theory rests upon the unification of the Einstein, Newton, and Coulomb superforces:

$$\frac{c^4}{G_d} \left( = \frac{m_* c^2}{r_*} \right) = \frac{m_*^2 G}{r_*^2} = \frac{e_*^2}{r_*^2} \rightarrow r_* m_* c = \frac{e_*^2}{c} \quad (= \hbar) \quad (1)$$

where the ratio  $c^4/G_d$  is the curvature superforce that appears in the Einstein field equations.  $G$  is Newton's gravitational constant,  $c$  is the speed of light,  $m_*$  and  $r_*$  are the Planck mass and length respectively [5, p.1234], and  $e_*$  is the massless bare (or coupling) charge. The Planck time is  $t_* = r_*/c$  [5, p.1233]. The fine structure constant is given by the ratio  $\alpha \equiv e^2/e_*^2$ , where  $e$  is the observed electronic charge magnitude. The ratio  $e^2/c$  to the right of the arrow is the spin coefficient for the Planck particle (PP), the proton, and the electron cores, where  $\hbar$  is the reduced Planck constant. One of the  $e_*$ s in  $e_*^2$  belongs to the PP under consideration and the other to any one of the remaining PPs in the PV state.

The electron, proton, and PP Dirac cores associated with the PV theory defined above are

$$(\pm e_*, m_e) \quad (\pm e_*, m_p) \quad \text{and} \quad (\pm e_*, m_*) \quad (2)$$

respectively. The  $\pm$  signs in the equations include the antiparticles. Their coupling to the highly energetic PV state is through the spin equations

$$r_e m_e c = r_p m_p c = \frac{e_*^2}{c} = r_* m_* c. \quad (3)$$

The spin is generated in the zero-point PV oscillations [6].

All of the preceding equations are fixed in the sense that their structure is determined at the high PV energy level. This level is roughly nineteen (proton cores) to twenty-two (electron cores) orders-of-magnitude more energetic than the processes taking place at the electron or proton levels. The masses  $m_e$

and  $m_p$  are assumed to be created along with their Compton radii  $r_e$  and  $r_p$  from (3).

## II. NONRELATIVISTIC SCHRÖDINGER EQUATION

The PV state consists of a continuum that is pervaded by a degenerate collection of PP cores. Since the separate PPs in that state are roughly confined to a sphere of radius  $r_*$ , they are nonrelativistic [6].

In the PV theory the well-known energy and momentum operators take the form

$$\hat{E} = i\hbar \frac{\partial}{\partial t} = i e_*^2 \frac{\partial}{c \partial t} \quad (4)$$

and

$$\hat{p} = -i\hbar \nabla = -i \frac{e_*^2}{c} \nabla \quad (5)$$

with the aid of equation (1).

Using (4) and (5), the Schrödinger equation [7, p.21] with the wavefunction  $\psi$  can now be expressed as

$$\begin{aligned} (\hat{E} - H) \psi &= \left( i e_*^2 \frac{\partial}{c \partial t} - H \right) \psi = \left( i e_*^2 \frac{\partial}{c \partial t} - \frac{\hat{p}^2}{2m_e} \right) \psi \\ &= \left( i e_*^2 \frac{\partial}{c \partial t} - \frac{e_*^4 \nabla^2}{2m_e c^2} \right) \psi = \frac{e_*^2}{c} \left( i \frac{\partial}{\partial t} - \frac{e_*^2 \nabla^2}{2m_e c} \right) \psi \\ &= \frac{e_*^2}{c} \left( i \frac{\partial}{\partial t} - \frac{r_e^2 \nabla^2}{2t_e} \right) \psi = 0 \end{aligned} \quad (6)$$

where again the 2 comes from the fact that the energy of the PPs within the PV state are nonrelativistic. The electron time constant  $t_e (= r_e/c)$  yields the distance  $r_e$  a photon travels in  $t_e$  seconds. The conjugate square for  $\hat{p}$  is used above.

## III. CONCLUSIONS AND COMMENTS

The equation from (6)

$$\frac{e_*^2}{c} \left( i \frac{\partial}{\partial t} - \frac{r_e^2 \nabla^2}{2t_e} \right) \psi = 0 \quad (7)$$

is the Schrödinger equation as derived in the PV theory. The parenthesis in equation (7) yields

$$i \frac{\partial \psi}{\partial t} = \frac{r_e^2 \nabla^2}{2t_e} \psi \quad (8)$$

with the formal solution

$$\psi = \exp \left[ i \frac{r_e^2 \nabla^2}{2t_e} t \right]. \quad (9)$$

The  $H$  operator for (7) now becomes

$$H = \frac{e_*^2}{c} \left( \frac{r_e^2 \nabla^2}{2t_e} \right) \quad (10)$$

with its spin coefficient  $e_*^2/c$  intact.

The quantum field solution in equation (A5) for the  $n$ th interval from the time-ordering operator in the quantum field theory (Appendix A) is

$$\begin{aligned} & \int_t^{t_{n-1}} c dt_n \frac{H(t_n)}{e_*^2} \\ &= \frac{(-i)^n}{n!} \int_t^{t_{n-1}} dt_n \left( \frac{r_e^2 \nabla^2}{2t_e} \right). \end{aligned} \quad (11)$$

Comparing equations (10) and (11) shows that equation (7) contains particle spin ( $e_*^2/c$ ) while the right side of (11) does not. This difference is to be expected as (11) describes a field rather than a particle core.

#### APPENDIX A TIME-ORDERING OPERATOR T

The time-ordering operator as given in the quantum field theory [8, p.306] is defined by

$$T \exp \left[ -i \int_t^{t'} dt'' H(t'')/\hbar \right] \equiv \quad (A1)$$

$$\sum_{n=0}^{\infty} \frac{1}{n!} \left( \frac{-i}{\hbar} \right)^n \int_t^{t'} dt_1 \cdots \int_t^{t_{n-1}} dt_n H(t_1) \cdots H(t_n) \quad (A2)$$

$$= \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int_t^{t'} dt_1 \cdots \int_t^{t_{n-1}} dt_n H'(t_1) \cdots H'(t_n) \quad (A3)$$

where from (3)

$$H'(t) = \frac{H(t)}{e_*^2/c} = \frac{H(t)}{r_e m_e c} = \frac{H(t)}{r_p m_p c} = \frac{H(t)}{r_* m_* c}. \quad (A4)$$

From (A3) and (10), the  $n$ th term leads to

$$\begin{aligned} & \int_t^{t_{n-1}} c dt_n \frac{H(t_n)}{e_*^2} \\ &= \frac{(-i)^n}{n!} \int_t^{t_{n-1}} dt_n \left( \frac{r_e^2 \nabla^2}{2t_e} \right) \end{aligned} \quad (A5)$$

for a quantum field without spin.

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