

Reducing the Covariant Dirac Equation for the Electron or Proton Cores to the Gradients of a Single Wavefunction

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Abstract—The particle- and antiparticle-equations for the electron or proton cores are coupled Dirac equations, each of which contains two wavefunctions. Without solving for the wavefunctions explicitly, it is possible to decouple and solve the equations exactly and completely for the electron and proton cores in terms of a single wavefunctions' gradients.

Calculations repeatedly demonstrate that the reduced Planck constant represents a particle or antiparticle spin coefficient.

Index Terms—Electron- and Proton-Core Wavefunctions, Planck Vacuum Theory.

I. INTRODUCTION

THE Planck vacuum (PV) theory supports a 7-dimensional (7D) spacetime that consists of two 4-dimensional (4D) spacetimes, an observed 4D spacetime that contains the particles and an unobserved 4D spacetime that contains the antiparticles. The separate 4D spacetimes prevent the particles and antiparticles from annihilating one another. The following calculations expand on this arrangement.

The theoretical foundation [1] [2] [3] [4] of the PV theory rests upon the unification of the Einstein, Newton, and Coulomb superforces:

$$\frac{c^4}{G} \left(= \frac{m_* c^2}{r_*} \right) = \frac{m_*^2 G}{r_*^2} = \frac{e_*^2}{r_*^2} \rightarrow r_* m_* c = \frac{e_*^2}{c} \quad (= \hbar) \quad (1)$$

where the ratio c^4/G is the curvature superforce that appears in the Einstein field equations. G is Newton's gravitational constant, m_* and r_* are the Planck mass and length respectively [5, p.1234], and e_* is the massless bare (or coupling) charge. The fine structure constant is given by the ratio $\alpha \equiv e^2/e_*^2$, where e is the observed electronic charge magnitude. The ratio e_*^2/c to the right of the arrow is the spin coefficient for the Planck particle (PP), the proton, and the electron cores respectively, where \hbar is the reduced Planck constant.

The two particle/PV coupling forces

$$F_e(r) = \frac{e_*^2}{r^2} - \frac{m_e c^2}{r} \quad \text{and} \quad F_p(r) = \frac{e_*^2}{r^2} - \frac{m_p c^2}{r} \quad (2)$$

the electron core $(-e_*, m_e)$ and proton core $(+e_*, m_p)$ exert on the invisible PV state; along with their coupling constants

$$F_e(r_e) = 0 \quad \text{and} \quad F_p(r_p) = 0 \quad (3)$$

and the resulting Compton (coupling) radii

$$r_e = \frac{e_*^2/c}{m_e c} \quad \text{and} \quad r_p = \frac{e_*^2/c}{m_p c} \quad (4)$$

lead to the important string of Compton (coupling) relations

$$r_e m_e c = r_p m_p c = \frac{(\pm e_*)^2}{c} = r_* m_* c \quad (= \hbar) \quad (5)$$

for the electron and proton cores (and their antiparticles). The PP Compton radius is $r_* = e_*^2/m_* c^2$, derived by equating the Einstein and Coulomb superforces from (1). To reiterate, the equations in (2) represent the forces the free electron or proton cores exert on the invisible PV space, a space that is itself pervaded by a degenerate collection of PP cores $(\pm e_*, m_*)$. The positron and antiproton cores are $(+e_*, m_e)$ and $(-e_*, m_p)$ respectively.

The e_*^2/c on the left sides of (7)–(10) is the spin coefficient for the PP, the proton, and the electron cores of the PV theory. One of the e_* s in e_*^2 belongs to the free particle core and the other charge belongs to any one of the PP cores making up the degenerate PV state.

After the Introduction in Section I, Section II reviews the Dirac core equations for the electron and proton cores. Section III presents comments and conclusions.

II. DIRAC CORE WAVEFUNCTIONS

The 4x1 covariant Dirac equation

$$\left(i\hbar\gamma^\mu \frac{\partial}{\partial x^\mu} - mc \right) \psi = \left(i\frac{e_*^2}{c}\gamma^\mu \frac{\partial}{\partial x^\mu} - mc \right) \psi = 0 \quad (6)$$

leads to the following four 2x1 spinor equations for the u and v wavefunctions [6], (where $x^0 = ct$ and the sum is over $j=1, 2, 3$)

$$i\frac{e_*^2}{c}(u', v') = m_e c u' \quad (7)$$

$$-i\frac{e_*^2}{c}(v', u') = m_e c v' \quad (8)$$

and

$$i\frac{e_*^2}{c}(u'', v'') = m_p c u'' \quad (9)$$

$$-i\frac{e_*^2}{c}(v'', u'') = m_p c v'' \quad (10)$$

which, from top to bottom, describe the electron, positron, proton, and antiproton cores respectively. The u s and v s are the 2x1 spinor wavefunction solutions to the equations,

including the spin coefficient e_*^2/c on the left and the various spin momenta $m_e c$ and $m_p c$ on the right. Equations (7)–(10) are *spin angular-momentum equations*, where the gradient operators on the left are defined in Appendix A. The form of these equations is a direct response of the vacuum state to the free electron or proton coupling forces in (2).

The ratio e_*^2/c is the spin coefficient, where

$$\vec{S} = \frac{e_*^2}{c} \vec{\sigma} \quad \longrightarrow \quad \frac{e_*^2}{c} \sigma_j \frac{\partial}{\partial x^j} \quad (11)$$

is the relativistic spin of the electron or proton cores. The 2x2 Pauli spin vector is $\vec{\sigma}$. The second expression is the scalar-product sum of \vec{S} with the gradient operator $\partial/\partial x^j$; that is, the PV gradient $\partial/\partial x^j$ in the j th direction weighted by the relativistic spin in that direction. Both equations are 2x2 matrix equations. As seen from the $(\pm e_*)^2$ in (5): *the spin magnitudes of the PP, the proton, and the electron cores are identical (the spin of their antiparticles is the negative of these)*.

Annihilation, i.e. adding the separate components [$e_*^2/c, u, v, m_e, m_p$] from the electron and proton equations (7)–(10), leads to:

$$(7) \oplus (8) =$$

$$i \overbrace{\left(\frac{e_*^2}{c} - \frac{e_*^2}{c} \right)}^0 (u' + v', v' + u') = m_e c (u' + v') \quad (12)$$

for the electron-positron, and

$$(9) \oplus (10) =$$

$$i \overbrace{\left(\frac{e_*^2}{c} - \frac{e_*^2}{c} \right)}^0 (u'' + v'', v'' + u'') = m_p c (u'' + v'') \quad (13)$$

for the proton-antiproton, where

$$(u' + v') = \hat{0} \quad \text{and} \quad (u'' + v'') = \hat{0} \quad (14)$$

are the 2x1 null solutions to (12) and (13). *Equations (12) and (13) constitute the electron and proton annihilation equations in the PV theory—reflecting the experimental fact that the core and anticore form a particle-antiparticle pair.*

III. COMMENTS AND CONCLUSIONS

Using Appendix A, the uv **coupled** equations (7)&(8) or (9)&(10) can be expressed as

$$i \frac{e_*^2}{c} \left(\frac{\partial u}{\partial x^0} + \sigma_j \frac{\partial v}{\partial x^j} \right) = m_e c u \quad (15)$$

and

$$-i \frac{e_*^2}{c} \left(\frac{\partial v}{\partial x^0} + \sigma_j \frac{\partial u}{\partial x^j} \right) = m_e c v. \quad (16)$$

Dividing (15) and (16) by the momentum $m_e c$ gives

$$i r_e \left(\frac{\partial u}{\partial x^0} + \sigma_j \frac{\partial v}{\partial x^j} \right) = u \quad (17)$$

and

$$-i r_e \left(\frac{\partial v}{\partial x^0} + \sigma_j \frac{\partial u}{\partial x^j} \right) = v \quad (18)$$

where now u and v and r_e represent both the electron and proton cores. Substituting $v = -u$ and $u = -v$ from (14) into (17)&(18) yields

$$i r_e \left(\frac{\partial u}{\partial x^0} - \sigma_j \frac{\partial u}{\partial x^j} \right) = u \quad (19)$$

and

$$-i r_e \left(\frac{\partial v}{\partial x^0} - \sigma_j \frac{\partial v}{\partial x^j} \right) = v \quad (20)$$

for the **decoupled** version of (7)&(8) that are now dependent only upon the single wavefunction gradient of u or v . A very rough idea concerning the nature of the solutions in (19)&(20) can be had by looking at the temporal components and treating u or v as simple functions rather than 2x1 spinors: then

$$i r_e \frac{\partial u}{\partial x^0} = u \quad \iff \quad u = \exp(-i x^0 / r_e). \quad (21)$$

Finally, using $\partial_j \equiv \partial/\partial x^j$ and (B4) for the spatial derivative

$$\sigma_j \frac{\partial u}{\partial x^j} = \sigma_j \partial_j u = \sigma_1 \partial_1 u + \sigma_2 \partial_2 u + \sigma_3 \partial_3 u$$

or

$$\sigma_j \partial_j u = \partial_1 \begin{pmatrix} u_2 \\ u_1 \end{pmatrix} + i \partial_2 \begin{pmatrix} -u_2 \\ u_1 \end{pmatrix} + \partial_3 \begin{pmatrix} +u_1 \\ -u_2 \end{pmatrix} \quad (22)$$

where the upper and lower components of u are u_1 and u_2 respectively. The first two terms in (22) are concerned with particle spin in the (12) or (xy) plane. The third term concerns the spin axis that points in the (3) or (z) direction, where the + or – signs refer to clockwise or counter-clockwise rotations of the axis or *visa versa*.

APPENDIX A GRADIENT OPERATOR

The gradient operator (summing over $j = 1, 2, 3$)

$$(U, V) \equiv \left(\frac{\partial U}{\partial x^0} + \sigma_j \frac{\partial V}{\partial x^j} \right) \quad (A1)$$

is defined for equations of the form in (7)–(10) and their wavefunctions U and V .

APPENDIX B THE γ AND β MATRICES

The 4x4 γ and β matrices used in the Dirac theory are defined here [7, p.91]: where

$$\gamma^0 \equiv \beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \quad (B1)$$

and ($j = 1, 2, 3$)

$$\gamma^j \equiv \beta \alpha_j = \begin{pmatrix} 0 & \sigma_j \\ -\sigma_j & 0 \end{pmatrix} \quad (B2)$$

and where I is the 2x2 unit matrix and

$$\alpha_j = \begin{pmatrix} 0 & \sigma_j \\ \sigma_j & 0 \end{pmatrix} \quad (B3)$$

where the σ_j are the 2x2 Pauli spin matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (B4)$$

The zeros in (B1)–(B3) and (B5) are 2x2 null matrices.

The 0 on the right side of (6) represents the 4x4 null matrix and

$$mc \equiv mc \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix}. \quad (\text{B5})$$

The wavefunction ψ is a 4x1 spinor matrix.

The coordinates x^μ are

$$x^\mu = (x^0, x^1, x^2, x^3) \quad (\text{B6})$$

where $x^0 \equiv ct$.

REFERENCES

- [1] P. Davies, *Superforce: the Search for a Grand Unified Theory of Nature*. Simon and Schuster, Inc., New York, 1984.
- [2] W.C. Daywitt. A Model for Davies' Universal Superforce. Galilean Electrodynamics, Sept./Oct., 83, 2006. (see also www.planckvacuumDOTcom)
- [3] W.C. Daywitt. The Trouble with the Equations of Modern Fundamental Physics. American Journal of Modern Physics. Special Issue: "Physics without Higgs and without Supersymmetry", 2016;5(1-1):22.
- [4] W.C. Daywitt, Comparing the Planck-Vacuum and the Urantia-Book Depictions of the Seven-Dimensional Spacetime, European Journal of Engineering Research and Science, 2020;5(12).
- [5] B.W. Carroll, D.A. Ostlie, *An Introduction to Modern Astrophysics*. (Addison-Wesley, San Francisco, Boston, New York, Cape Town, Hong Kong, London, Madrid, Mexico City, Montreal, Munich, Paris, Singapore, Sidney, Tokyo, Toronto, 2007).
- [6] W.C. Daywitt, The Seven Dimensional Spacetime in the Planck Vacuum Theory and the Structure of the Electron and Proton Cores. European Journal of Engineering Research and Science, 2020;5(4).
- [7] D.M. Gingrich, *Practical Quantum Electrodynamics*. (CRC, The Taylor & Francis Group, Boca Raton, London, New York, 2006).