A Modern View of the Bohr Hydrogen Atom and Its Coupling to the Vacuum State Leads to Simple Expressions for the Fine Structure Constant, the Rydberg Constant, and Particle Spin

William C. Daywitt

National Institute for Standards and Technology (retired), Boulder, Colorado. E-mail: wcdaywitt@me.com

This paper argues that the coupling of the electron and proton cores (in the Bohr hydrogen atom) to the Planck vacuum (PV) state is the source of the fine structure constant, as that constant provides the connection between the charge of the cores and the electronic charge measured in the laboratory. The use of the Bohr atom in the calculations rids the results of electron or proton structure considerations, since the particles in the calculations are massive point charges. The fine structure constant and particle spin emerge as characteristic features of the Bohr-atom coupling to the vacuum state.

key words–**Fundamental Physics, Bohr Hydrogen Atom, Coupling Charge, Fine Structure Constant, Particle Spin.**

"beyond the standard model"

1 Introduction

We open with the following quote [1, p.129] from Feynman: "There is a most profound and beautiful question associated with the coupling constant, e—the amplitude for a real electron to emit or absorb a real photon. It is a simple number that has been experimentally determined to be close to -0.08542455. (My physicist friends won't recognize this number, because thy like to remember it as the inverse of its square: about 137.03597 with an uncertainty of about 2 in the last decimal place. It [the fine structure constant] has been a mystery ever since it was first discovered more than fifty years ago, and all good theoretical physicists put this number up on their wall and worry about it.)"

The theoretical foundation [2] [3] [4] of the PV theory rests upon the unification of the Einstein, Newton, and Coulomb superforces:

$$\frac{c^4}{G}\left(=\frac{m_*c^2}{r_*}\right)=\frac{m_*^2G}{r_*^2}=\frac{e_*^2}{r_*^2} \tag{1}$$

where the ratio c^4/G is the curvature superforce that appears in the Einstein field equations. G is Newton's gravitational constant, c is the speed of light, m_* and r_* are the Planck mass and length respectively [5, p.1234], and e_* is the massless bare (or coupling) charge. The fine structure constant is given by the ratio $\alpha \equiv e^2/e_*^2$, where e is the observed electronic charge.

The two particle/PV coupling forces

$$F_e(r)=\frac{e_*^2}{r^2}-\frac{m_ec^2}{r} \quad \text{and} \quad F_p(r)=\frac{e_*^2}{r^2}-\frac{m_pc^2}{r} \tag{2}$$

the electron core $(-e_*, m_e)$ and the proton core (e_*, m_p) exert on the invisible PV state; along with their coupling constants

$$F_e(r_e)=0 \quad \text{and} \quad F_p(r_p)=0 \tag{3}$$

and the resulting Compton radii

$$r_e=\frac{e_*^2}{m_ec^2} \quad \text{and} \quad r_p=\frac{e_*^2}{m_pc^2} \tag{4}$$

lead to the important string of Compton relations

$$r_em_ec^2=r_pm_pc^2=e_*^2=r_*m_*c^2 \quad (=c\hbar) \tag{5}$$

for the electron and proton cores, where $\hbar$ is the reduced Planck constant. The Compton relation on the right side of e_*^2 comes from equating the Einstein and Coulomb superforces from (1). To reiterate, the equations in (2) represent the forces the free electron and proton cores exert on the invisible PV space, a space that is itself pervaded by a degenerate collection of Planck-particle cores $(\pm e_*, m_*)$ [6].

The Lorentz invariance of the coupling constants in (3) leads to the energy and momentum operators of the quantum theory [6].

2 Bohr Hydrogen Atom

At this point the nomenclature of the previous section (with its undergrowth of subscripts) is changed to a simpler notation.

The Bohr atom [7] [8, p.73] [Appendix A] consists of a massive point-electron $(-e, m) = (-\alpha^{1/2}e_*, m)$ and a massive point-proton $(e, M) = (\alpha^{1/2} e_*, M)$ circling their common center of mass. Their masses and circling radii are related by the equation $mr = MR$, where r and R in this equation (and in what follows) represent these radii. The r here IS NOT the same as the coordinate radius r in equations (2). The form of the massive charges for the electron and proton, and the fine structure constant, insure that the squared charges e^2 and αe_*^2 can be interchanged in what follows. *It will be seen below that the quantum mechanical equivalent of $mr = MR$ is $m_e r_e = m_p r_p$.*

The quantized angular momentum assumed by Bohr is given by

$$(mr^2 + MR^2)\omega = mr^2\omega(1 + m/M) = n\hbar \tag{6}$$

in terms of the Planck constant $\hbar$. The centripetal force driving the electron toward the center of mass is given by

$$m\omega^2 r = \frac{e^2}{(r+R)^2} = \frac{e^2}{r^2(1+m/M)^2} \tag{7}$$

and the total energy $T + V$ by (using (7))

$$\begin{aligned} E &= \frac{mr^2\omega^2}{2} + \frac{MR^2\omega^2}{2} - \frac{e^2}{r+R} \\ &= -\frac{mr^2\omega^2}{2}(1 + m/M) \end{aligned} \tag{8}$$

where ω is the angular frequency of the electron and proton about their common center of mass.

Each of the equations (6)–(8) may be solved for $\omega^2 r^4$:

$$\begin{aligned} \omega^2 r^4 &= \frac{n^2\hbar^2}{m^2(1+m/M)^2} = A \\ &= \frac{e^2 r}{m(1+m/M)^2} = rB \\ &= -\frac{2Er^2}{m(1+m/M)} = r^2 EC \end{aligned} \tag{9}$$

where $m \equiv m_e$ and $M \equiv m_p$. Eliminating r from the right sides of the above equalities then leads to the energy spectrum $(n = 1, 2, 3, \ldots)$ [8, p.74, footnote]

$$E_n = \frac{B^2}{AC} = -\frac{me^4}{2n^2\hbar^2(1+m/M)} \tag{10}$$

and the electron-orbit radii

$$r_n = \frac{A}{B} = \frac{n^2\hbar^2}{me^2}\,. \tag{11}$$

The wavenumber (number of waves per centimeter) for the radiation between the energy states n and n' is given by

$$\frac{\nu}{c} = \frac{E_n - E_{n'}}{2\pi c\hbar} = R_H\left(\frac{1}{n'^2} - \frac{1}{n^2}\right) \tag{12}$$

where the Rydberg constant is

$$R_H = \frac{me^4}{4\pi c\hbar^3(1+m/M)}\,. \tag{13}$$

These results all depend upon the single *secondary* constant $\hbar$; so their fundamental physical meaning is obscured. The next section remedies this shortcoming.

3 Planck Vacuum Perspective

This section replaces the single constant $\hbar$ of the previous section by the *primary* (fundamental) constants from the PV theory given in the Introduction. It is left for the reader to fill-in the simple steps. The new results due to the PV theory pre-multiply the bracketed quantities in the following equations.

Using (5), the new angular momentum from (6) becomes $(\epsilon \equiv m/M)$

$$\begin{aligned} (mr^2 + MR^2)\omega &= mr^2\omega(1+\epsilon) \\ &= [n\hbar] = \frac{e_*^2}{c}[n] = r_e mc[n]\,. \end{aligned} \tag{14}$$

The squared charge $e_*^2 = (-e_*)(-e_*)$ in the first equation of (2) represents the numerator of the Coulomb force the electron core (the first charge) exerts on the separate Planck-particle cores (the second charge) of the PV state. As such, its appearance in (14) is a manifestation of that coupling force. The $r_e mc$ in (14) comes from the Compton relations in (5). Thus, from the first equation in (3), the quantized angular momentum is restricted to those values $(r_e mc)$ where the electron coupling force $F_e(r_e)$ acting on the vacuum state vanishes, and there are a denumerable infinity $(n = 1, 2, 3, \ldots)$ of such values. It is noted in an earlier paper [9] that *the ratio e_*^2/c is the spin coefficient for the electron and proton cores.*

The new energy levels are given by

$$E_n = -\alpha^2 mc^2\left[\frac{1}{2n^2(1+\epsilon)}\right] \tag{15}$$

where mc^2 is the mass energy of the electron and α is the fine structure constant [8, p.200].

The new Bohr electron orbits are

$$r_n = \frac{r_e}{\alpha}[n^2] \tag{16}$$

where $r_e (= e_*^2/m_e c^2)$ is the electron Compton radius. This result yields the correct value for r_1 with e_* in cgs-esu units [8, p.722].

The energy/orbit ratios for the electron are

$$\frac{E_n}{r_n} = -\alpha^3\frac{mc^2}{r_e}\left[\frac{1}{2n^4(1+\epsilon)}\right] \tag{17}$$

$$= -\alpha^3 \frac{m_* c^2}{r_*} n_{re} \left[\frac{1}{2n^4(1+\epsilon)} \right] \tag{18}$$

where

$$n_{re} \equiv \frac{mc^2/r_e}{m_* c^2/r_*} \tag{19}$$

is the n-ratio that appears in the Schwarzschild line element for the Einstein field equations [10].

The new hydrogen-atom Rydberg constant is given by

$$R_H = \frac{\alpha^2}{r_e} \left[\frac{1}{4\pi(1+\epsilon)} \right] . \tag{20}$$

This result yields the correct value for the Rydberg constant in cgs-esu units [8, p.723].

4 Conclusions and Comments

Since the angular momentum has been quantized, the classical variables that are part of the analysis take on new meanings. For example, from (14) the angular frequency becomes

$$\omega_n = \frac{r_e mc[n]}{m r_n^2 (1+\epsilon)} = \alpha \frac{\alpha c}{r_e} \left[\frac{1}{(1+\epsilon) n^3} \right]$$

$$\approx \alpha \frac{\alpha c}{r_e} \left[\frac{1}{n^3} \right] \tag{21}$$

where, from (23), αc is the velocity of the electron in the first ($n = 1$) Bohr orbit around the center of mass.

Using (21) for ω, the centripetal acceleration becomes

$$\omega_n^2 r_n = \alpha \frac{\alpha^2 c^2}{r_e} \left[\frac{1}{(1+\epsilon)^2 n^4)} \right] \approx \alpha \frac{\alpha^2 c^2}{r_e} \left[\frac{1}{n^4} \right] \tag{22}$$

and the orbit velocity of the electron becomes

$$v_n = r_n \omega_n = \frac{r_e}{\alpha} [n^2] \cdot \alpha \frac{\alpha c}{r_e} \left[\frac{1}{(1+\epsilon) n^3} \right]$$

$$= \alpha c \left[\frac{1}{(1+\epsilon) n} \right] \approx \alpha c \left[\frac{1}{n} \right] . \tag{23}$$

The preceding results demonstrates that the fine structure constant is not a highly sophisticated constant: it is merely the squared-ratio of two important charge magnitudes, the observable electronic charge e and the unobserved coupling charge e_*. In particular, its derivation is independent of the radiative corrections [11, p.298] [9] from the quantum electrodynamics QED theory. Furthermore, it is noted that all of the pre-multipliers in equations (15)–(18) and (20) are particle/PV coupling parameters, *now including the fine structure constant.*

Finally, using the Feynman "amplitude e" at the beginning of the Introduction as a guide, defining the electron spin coefficient as e^2/c and using (5) then leads to the ratio

$$\left(\frac{\text{electron spin}}{\text{electron–core spin}} \right)_{\text{coeff}} = \frac{e^2/c}{r_e mc} = \frac{e^2/c}{e_*^2/c} = \frac{e^2}{e_*^2} = \alpha \tag{24}$$

where the *nominal* electron-core spin coefficient is $r_e mc$, while the *physical* quantity of interest is e_*^2/c.

Appendix A: The Bohr Atom

The Bohr theory [8, p.72], in its final form, embodied the following ideas:

1. A hydrogen atom consists of a positive nucleus (proton) and a single electron in a state of relative circular motion under the action of their mutual electrical attraction.

2. The atom may remain for extended periods of time in a given state *without radiating electromagnetic waves, provided this state is one for which the angular momentum of the atom is an integral multiple n of ℏ.*

3. Radiation is emitted whenever the atom "jumps" from one of the "allowed" states of energy E_1 to another of lower energy E_2.

4. When radiation is emitted, its frequency ν is determined by the Einstein frequency condition

$$h\nu = E_1 - E_2 . \tag{A1}$$

References

[1] R.P. Feynman, *QED: THE STRANGE THEORY OF LIGHT AND MATTER.* Princeton University Press, Princeton and Oxford, UK, 1985.

[2] P. Davies, *Superforce: the Search for a Grand Unified Theory of Nature*, (Simon and Schuster, Inc., New York, 1984).

[3] W.C. Daywitt, *A Model for Davies' Universal Superforce*, Galilean Electrodynamics, vol.5, (Sept./Oct.), 83, (2006). See also www.planckvacuumDOTcom.

[4] W.C. Daywitt, "The Trouble with the Equations of Modern Fundamental Physics", American Journal of Modern Physics, Special Issue: Physics without Higgs and without Supersymmetry. vol.5, no.1-1, 22, (2016).

[5] B.W. Carroll, D.A. Ostlie, *An Introduction to Modern Astrophysics*, (Addison-Wesley, San Francisco, Boston, New York, Cape Town, Hong Kong, London, Madrid, Mexico City, Montreal, Munich, Paris, Singapore, Sydney, Tokyo, Toronto, 2007).

[6] W.C. Daywitt. The Planck Vacuum. Progress in Physics, v. 1, 20, 2009.

[7] N. Bohr, "On the Constitution of Atoms and Molecules", *Philos. Mag.*, **26**, 1, 1913.

[8] R.B. Leighton, *Principles of Modern Physics*, (McGraw-Hill Book Co., New York, Toronto, London, 1959). The present paper uses cgs-esu units; so the use of $e^2/4\pi\epsilon_0$ in this reference should be replaced by just e^2 for the present analysis. Also note the use of *both* $\hbar$ and h in pages 73 through 75.

[9] W.C. Daywitt, The Anomalous Magnetic Moment of the Electron and Proton Cores According to the Planck Vacuum Theory. European Journal of Engineering Research and Science, VOL. 4, NO.6, June 2019.

[10] W.C. Daywitt, "The Planck Vacuum and the Schwarzschild Metrics", Progress in Physics, vol.3, 30, (2009).

[11] D.M. Gingrich, *Practical Quantum Electrodynamics*, (CRC, The Taylor & Francis Group, Boca Raton, London, New York, 2006).