Spin and the Anomalous Magnetic Moment of the Dirac Particle

William C. Daywitt

National Institute for Standards and Technology (retired), Boulder, Colorado. E-mail: wcdaywitt@me.com

The Planck vacuum (PV) theory is a complete elementary-particle theory in the sense that it connects: the Bohr point-electron and point-proton; the Schrödinger electron; and the Dirac particle (electron, positron, proton, antiproton) to the PV state. The present paper shows: that the Dirac particle spin has its source in the electron and proton coupling forces to the PV state; why the electron and proton have the same spin, but different g-factors; and that the proton g-factor can be roughly estimated from the proton structure constant and the Schwinger QED estimate of the electron g-factor.

[Reference — www.planckvacuumDOTcom.]

1 Introduction

The theoretical foundation [1] [2] [3] of the PV theory rests upon the unification of the Einstein, Newton, and Coulomb superforces:

$$\frac{c^4}{G}\left(=\frac{m_*c^2}{r_*}\right)=\frac{m_*^2G}{r_*^2}=\frac{e_*^2}{r_*^2} \tag{1}$$

where the ratio c^4/G is the curvature superforce that appears in the Einstein field equations. G is Newton's gravitational constant, c is the speed of light, m_* and r_* are the Planck mass and length respectively [4, p.1234], and e_* is the massless bare charge. The fine structure constant is given by the ratio $\alpha=e^2/e_*^2$, where $(-e)$ is the observed electronic charge.

In the following text the terms electron, proton, and Planck-particle include their respective antiparticles. A Dirac particle is a particle that obeys the Dirac equation; and a "real particle" is a Dirac particle with radiative corrections. Also, whether a particle is a Dirac particle or a "real particle" is often left to the context in which the terms are used.

The two particle/PV coupling forces

$$F_e(r)=\frac{e_*^2}{r^2}-\frac{m_ec^2}{r} \quad \text{and} \quad F_p(r)=\frac{e_*^2}{r^2}-\frac{m_pc^2}{r} \tag{2}$$

the electron core $(-e_*,m_e)$ and proton core $(+e_*,m_p)$ exert on the invisible PV state; along with their coupling constants

$$F_e(r_e)=0 \quad \text{and} \quad F_p(r_p)=0 \tag{3}$$

and the resulting Compton radii

$$r_e=\frac{e_*^2}{m_ec^2} \quad \text{and} \quad r_p=\frac{e_*^2}{m_pc^2} \tag{4}$$

lead to the important string of Compton relations

$$r_em_ec^2=r_pm_pc^2=e_*^2=r_*m_*c^2 \quad (=c\hbar) \tag{5}$$

for the electron and proton cores, where $\hbar$ is the reduced Planck constant. The Planck particle Compton radius is $r_*=e_*^2/m_*c^2$. To reiterate, the equations in (2) represent the forces the free electron and proton cores exert on the invisible PV space, a space that is itself pervaded by a degenerate collection of Planck-particle cores $(-e_*,m_*)$ [5]. The positron and antiproton cores are $(+e_*,m_e)$ and $(-e_*,m_p)$ respectively.

Finally, the Lorentz invariance of the coupling constants in (3) leads to the energy $(i\hbar\partial/\partial t)$ and momentum $(-i\hbar\nabla)$ operators of the quantum theory [5].

The next section derives the Dirac particle spin and shows that the spin has its source in the PV Compton relations. Section 3 derives the magnetic moments and g-factor for the Dirac particle, and Section 4 makes a rough estimate of the proton g-factor from the proton structure constant and the Schwinger g-factor for the electron coupled to a magnetic field.

2 Dirac particle spin

Using (5), the manifestly covariant form [6, p.91] of the Dirac equation for the Dirac particle can be expressed as:

$$\left(i\hbar\gamma^\mu\frac{\partial}{\partial x^\mu}-mc\right)\psi=\left(i\frac{e_*^2}{c}\gamma^\mu\frac{\partial}{\partial x^\mu}-mc\right)\psi \tag{6}$$

$$=\left[i\frac{e_*^2}{c}\gamma^0\frac{\partial}{\partial x^0}+i\begin{pmatrix}0 & S_j\\ -S_j & 0\end{pmatrix}\frac{\partial}{\partial x^j}-mc\right]\psi=0 \tag{7}$$

where the second term in (7) is summed over $j=1,2,3$ and

$$\begin{pmatrix}0 & S_j\\ -S_j & 0\end{pmatrix}=\begin{pmatrix}0 & e_*^2\sigma_j/c\\ -e_*^2\sigma_j/c & 0\end{pmatrix} \tag{8}$$

where one of the charges in e_*^2 belongs to the free particle and the other to any one of the Planck-particle cores within the degenerate PV state. The $e_*^2\sigma_j/c$ in the 4x4 matrix on the right side of (8) are the 2x2 spin components of the S-vector

$$\overrightarrow{S} = \frac{e_*^2}{c}\overrightarrow{\sigma} \;\; (= \hbar\overrightarrow{\sigma}) \tag{9}$$

that applies to all the Dirac particles. $\overrightarrow{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ is the Pauli spin vector, where the σ_js are 2x2 matrices. The spin operator of the Pauli equation associated with (9) is then [6, p.81]

$$\overrightarrow{s} = \frac{\overrightarrow{S}}{2} = \frac{1}{2}\frac{e_*^2}{c}\overrightarrow{\sigma} \;\; \left(= \frac{1}{2}\hbar\overrightarrow{\sigma}\right)$$

$$= \frac{r_e m_e c}{2}\overrightarrow{\sigma} = \frac{r_p m_p c}{2}\overrightarrow{\sigma} = \frac{r_* m_* c}{2}\overrightarrow{\sigma} \tag{10}$$

showing that the electron spin $[(r_e m_e c/2)\overrightarrow{\sigma}]$ and the proton spin $[(r_p m_p c/2)\overrightarrow{\sigma}]$ have the same value. Thus the source of the two spins are the electron and proton coupling constants in (3).

3 Gyromagnetic ratio g

For equations (6) and (7), the corresponding g-factor is exactly $g = 2$ [7, p.667]. This gyromagnetic ratio represents the magnetic/mechanical moment ratio (11) for the Dirac equation without radiative corrections.

In general, the intrinsic magnetic moment $\overrightarrow{\mu}$ is related to the spin vector through [6, p.81]

$$\overrightarrow{\mu} = g\mu_B\overrightarrow{s} \quad \longrightarrow \quad g\mu_B = \frac{\mu}{s} \tag{11}$$

where g is the Lande g-factor and μ_B is the Bohr magneton

$$\mu_B = \frac{e\hbar}{2m_e c} = \frac{ec\hbar}{2m_e c^2} = \frac{ee_*^2}{2m_e c^2} = \frac{er_e}{2} \tag{12}$$

where r_e is the electron Compton radius. Although the g-factor in (11) is exactly 2 for the Dirac equation, there is an anomalous-moment increase to this value due to radiative corrections applied to that equation [6, p.298].

Note that for the Dirac particle where $g = 2$, (11) yields

$$\overrightarrow{\mu} = er_e\overrightarrow{s} \quad \longrightarrow \quad \frac{\mu}{s} = er_e\,. \tag{13}$$

This is an unacceptable result for the Dirac proton; so (11) is replaced here by

$$\overrightarrow{\mu} = g\mu_c\overrightarrow{s} \quad \longrightarrow \quad \frac{\mu}{s} = g\mu_c \tag{14}$$

where $\mu_c = er_e/2$ for the electron and $\mu_c = er_p/2$ for the proton. Thus the correct baseline moments, normalized by their spin, for the Dirac particle are given by (14), where

$$\frac{\mu_e}{s} = er_e \quad \text{and} \quad \frac{\mu_p}{s} = er_p \tag{15}$$

for $g = 2$.

4 Anomalous magnetic moment

When a magnetic field is coupled to the Dirac equation in (6) and (7), the photons from that field interact with the product e_*^2, leading to a small increase in the electron g-factor and a large increase in the proton g-factor. From the Schwinger QED calculations [8], the electron g-factor change (due to radiative corrections) amounts to

$$\frac{\delta\mu}{\mu} = \frac{g}{2} - 1 = \frac{e^2}{2\pi c\hbar} = \frac{e^2}{2\pi e_*^2} = \frac{\alpha}{2\pi} = 0.001162 \tag{16}$$

where $e^2 = \alpha e_*^2$ and α is the fine structure constant. This field interaction with the charge product implies that the field is coupling to both the electron core $(-e_*, m_e)$ and the PV state, and introduces the fine structure constant into the calculations. There are also higher-order α corrections to (16) [6, p.82] that are of no interest to the present development, where for convenience (16) is taken to be the total relative change in μ.

The electron is thought to be a true point particle [6, p.82] because it contains no internal structure, as does the proton [9]. In the present context, however, it is appropriate to associate the "size" of the electron and proton with their Compton radii, where the corresponding proton structure constant (r_e/r_p) is defined by [10]

$$m_p = \frac{r_e}{r_p}m_e \quad \longrightarrow \quad \left(\frac{r_e}{r_p}\right) = \frac{m_p}{m_e} \approx 1836 \tag{17}$$

which derives from both the e_*^2 and m in (6). This suggests that the proton g-factor can be roughly estimated from

$$g_p \sim 2[1 + 0.001162(r_e/r_p)] = 2(1 + 2.13) \tag{18}$$

where the experimental value is [6, p.82]

$$(g_p)_{exp} = 2(1 + 1.79)\,. \tag{19}$$

As rough as it is, this estimate suggests that the PV parameters describing the Dirac particle carry over into the QED calculations for the anomalous magnetic moment of the radiative-corrected Dirac particle.

Submitted on October mm, 2018

References

[1] Davies P. Superforce: the Search for a Grand Unified Theory of Nature. Simon and Schuster, Inc., New York, 1984.

[2] Daywitt W.C. A Model for Davies' Universal Superforce. Galilean Electrodynamics, vol.5, (2006) (Sept./Oct.), 83. See also www.planckvacuumDOTcom.

[3] Daywitt W.C. The Trouble with the Equations of Modern Fundamental Physics. American Journal of Modern Physics. Special Issue: Physics without Higgs and without Supersymmetry. v. 5, (2016), no.1-1, 22.

[4] Carroll B.W., Ostlie D.A. An Introduction to Modern Astrophysics. Addison-Wesley, San Francisco, Boston, New York, Cape Town, Hong Kong, London, Madrid, Mexico City, Montreal, Munich, Paris, Singapore, Sydney, Tokyo, Toronto, 2007.

[5] Daywitt W.C. The Planck Vacuum. Progress in Physics, v. 1, 20, 2009.

[6] Gingrich D.M. Practical Quantum Electrodynamics. CRC, The Taylor & Francis Group, Boca Raton, London, New York, 2006.

[7] Leighton R.B. Principles of Modern Physics. McGraw-Hill Book Co., New York, Toranto, London, 1959.

[8] Schwinger, J. On Quantum-Electrodynamics and the Magnetic Moment of the Electron. Physical Review, 1948, Vol.73, p.416. Julian Schwinger won the Nobel Prize in 1965 for his work in QED.

[9] Daywitt W.C. A Planck Vacuum Pilot Model for Inelastic Electron-Proton Scattering. Progress in Physics, v. 11, Issue 4 (October), 308, 2015.

[10] Daywitt W.C. The Nature of the Electron and Proton as Viewed in the Planck Vacuum Theory. Progress in Physics, Issue 4, v. 14, 204, 2018.