

The Anomalous Magnetic Moment of the Electron and Proton Cores According to the Planck Vacuum Theory

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Abstract—Despite the resounding success of the quantum electrodynamic (QED) calculations, there remains some confusion concerning the Dirac equation's part in the calculation of the anomalous magnetic moment of the electron and proton. The confusion resides in the nature of the Dirac equation, the fine structure constant, and the relationship between the two. This paper argues that the Dirac equation describes the coupling of the electron or proton cores to the invisible Planck vacuum (PV) state; and that the fine structure constant connects that equation to the electron or proton particles measured in the laboratory.

Index Terms—Fundamental physics, electron, proton.

I. INTRODUCTION

IN the arena of elementary particle physics, the PV theory is a quasi-particle theory rather than a quantum field theory, where the electron and proton cores are particle-like objects (consisting of a massive charge) that are coupled to an invisible vacuum state. It is this particle-like aspect of the cores that is exploited in the following PV analysis of the anomalous magnetic moment of the electron and proton.

The theoretical foundation [1] [2] [3] of the PV theory rests upon the unification of the Einstein, Newton, and Coulomb superforces:

$$\frac{c^4}{G} \left(= \frac{m_* c^2}{r_*} \right) = \frac{m_*^2 G}{r_*^2} = \frac{e_*^2}{r_*^2} \quad (1)$$

where the ratio c^4/G is the curvature superforce that appears in the Einstein field equations. G is Newton's gravitational constant, c is the speed of light, m_* and r_* are the Planck mass and length respectively [4, p.1234], and e_* is the bare (or coupling) charge. The fine structure constant is $\alpha \equiv e^2/e_*^2$.

The two particle/PV coupling forces

$$F_e(r) = \frac{e_*^2}{r^2} - \frac{m_e c^2}{r} \quad \text{and} \quad F_p(r) = \frac{e_*^2}{r^2} - \frac{m_p c^2}{r} \quad (2)$$

the electron core ($-e_*, m_e$) and proton core ($+e_*, m_p$) exert on the invisible PV state; along with their coupling constants

$$F_e(r_e) = 0 \quad \text{and} \quad F_p(r_p) = 0 \quad (3)$$

and the resulting Compton radii

$$r_e = \frac{e_*^2}{m_e c^2} \quad \text{and} \quad r_p = \frac{e_*^2}{m_p c^2} \quad (4)$$

lead to the important string of Compton relations

$$r_e m_e c^2 = r_p m_p c^2 = e_*^2 = r_* m_* c^2 \quad (= c\hbar) \quad (5)$$

for the electron and proton cores, where \hbar is the reduced Planck constant. The Planck particle Compton radius is $r_* = e_*^2/m_* c^2$, which is derived by equating the Einstein and Coulomb superforces from (1). To reiterate, the equations in (2) represent the forces the free electron or proton cores exert on the invisible PV space, a space that is itself pervaded by a degenerate collection of Planck-particle cores ($\pm e_*, m_*$) [5]. The positron and antiproton cores are ($+e_*, m_e$) and ($-e_*, m_p$) respectively. It should be noted that all of the constants in (5) are Lorentz invariant.

Finally, the Lorentz invariance of the coupling constants in (3) lead to the energy

$$i\hbar \frac{\partial}{\partial t} = i e_*^2 \frac{\partial}{\partial ct} \quad (6)$$

and momentum

$$-i\hbar \nabla = -i \frac{e_*^2}{c} \nabla \quad (7)$$

operators of the quantum theory [5]. The two operators are proportional to the squared coupling charge e_*^2 , or to the spin coefficient e_*^2/c . The structure of these two operators is responsible for the structure of the gradients in (8) and (9), including the "time gradient" in the first term of (9).

Section 2 expresses the covariant Dirac equation in terms of PV parameters. Section 3 discusses the gyromagnetic g-factor. Section 4 discusses the electron g-factor and Section 5, the proton g-factor. Sections 4 and 5 represent a work in progress that seeks to relate the QED radiative corrections [6, p.201] to the PV coupling model. Section 6 presents some comments and conclusions.

II. DIRAC EQUATION

The Dirac particles defined in the present paper are the electron and proton *cores* and their antiparticles, where the following Dirac equations are assumed to result from the equations in (2) operating on the PV state. This assumption follows from the appearance of e_*^2 and m in the following Dirac equations, where m is the electron or proton mass.

Using (5), the manifestly covariant form [6, p.90] [Appendix A] of the Dirac equation for the the electron and proton cores can be expressed as:

$$\left(i c \hbar \gamma^\mu \frac{\partial}{\partial x^\mu} - m c^2 \right) \psi = \left(i e_*^2 \gamma^\mu \frac{\partial}{\partial x^\mu} - m c^2 \right) \psi = \quad (8)$$

$$\left[ie_*^2 \gamma^0 \frac{\partial}{\partial x^0} + i \begin{pmatrix} 0 & cS_j \\ -cS_j & 0 \end{pmatrix} \frac{\partial}{\partial x^j} - mc^2 \right] \psi = 0 \quad (9)$$

where the second term in (9) is summed over $j = 1, 2, 3$ and

$$\begin{pmatrix} 0 & cS_j \\ -cS_j & 0 \end{pmatrix} = \begin{pmatrix} 0 & e_*^2 \sigma_j \\ -e_*^2 \sigma_j & 0 \end{pmatrix} \quad (10)$$

where one of the charges in e_*^2 belongs to the free electron or proton cores, and the other to any one of the Planck-particle cores within the degenerate PV state. The $e_*^2 \sigma_j / c$ in the 4x4 matrix on the right side of (10) are the 2x2 spin components of the S-vector

$$\vec{S} = \frac{e_*^2}{c} \vec{\sigma} \quad (= \hbar \vec{\sigma}) \quad (11)$$

that applies to all the Dirac cores. $\vec{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ is the Pauli spin vector, where the σ_j s are 2x2 matrices.

Using the equations in (5), the fine structure constant can be expressed as

$$\alpha \equiv \frac{e^2}{e_*^2} = \frac{e^2}{r_* m_* c^2} = \frac{e^2}{r_p m_p c^2} = \frac{e^2}{r_e m_e c^2} \approx \frac{1}{137} \quad (12)$$

where e is the magnitude of the laboratory-observed electron or proton charge. If $e = e_*$, then the Compton relations in (5) yield $\alpha = 1$, implying that the fine structure constant provides the “bridge” over which the Dirac equation connects to the charge e .

III. GYROMAGNETIC RATIO

For (8) and (9) the g-factor is exactly $g = 2$ [7, p.667]. This gyromagnetic ratio represents the magnetic to mechanical moment ratio (13) for the Dirac equation without radiative corrections [6, p.298].

In general (radiative corrections or not), the intrinsic magnetic moment $\vec{\mu}$ is related to the spin vector $\vec{s} = \vec{S}/2$ through the equations [6, p.81]

$$\vec{\mu} = g \mu_B \vec{s} \quad \rightarrow \quad g \mu_B = \frac{\mu}{s} \quad (13)$$

where g is the g-factor and μ_B is the Bohr magneton

$$\mu_B = \frac{e \hbar}{2 m_e c} = \frac{e c \hbar}{2 m_e c^2} = \frac{e e_*^2}{2 m_e c^2} = \frac{e r_e}{2} \quad (14)$$

where r_e is the electron Compton radius. Although the g-factor in (13) is exactly 2 for the Dirac equation, there is an anomalous-moment increase to this value due to radiative corrections.

Note that for $g = 2$, (13) yields

$$\vec{\mu} = e r_e \vec{s} \quad \rightarrow \quad \frac{\mu}{s} = e r_e. \quad (15)$$

However, this is an unacceptable result for the Dirac proton core; so (13) is replaced here by

$$\vec{\mu} = g \mu_c \vec{s} \quad \rightarrow \quad \frac{\mu}{s} = g \mu_c \quad (16)$$

where $\mu_c = e r_e / 2$ for the electron and $\mu_c = e r_p / 2$ for the proton. Thus the correct baseline moments (no corrections) for the Dirac cores, normalized by their common spin, are given by (16) with $g = 2$, where

$$\frac{\mu_e}{s} = e r_e \quad \text{and} \quad \frac{\mu_p}{s} = e r_p \quad (17)$$

are the electron and proton magnetic dipole moments.

IV. ELECTRON G-FACTOR

When radiative corrections are included with (8) and (9), photon and Planck-particle scatterings taking place within the vacuum state lead to a small increase in the electron g-factor and a large increase in the proton g-factor. Using $\alpha^{-1} = 137.0$ [7, p.722] for the inverse fine structure constant in the Schwinger QED calculation [8][6, p.298], the relative change in the electron magnetic moment is

$$\begin{aligned} \frac{\delta \mu}{\mu} &= \frac{g}{2} - 1 = \frac{e^2}{2\pi c \hbar} = \frac{1}{2\pi} \frac{e^2}{e_*^2} \\ &= \frac{\alpha}{2\pi} = 0.001162 \end{aligned} \quad (18)$$

where one of the e_* s in the squared coupling charge e_*^2 belongs to the free electron core and the other to any one of the Planck-particle cores within the degenerate PV state.

The radiative correction for the electron core can be expressed as

$$\frac{\alpha}{2\pi} = \frac{e^2 / 2\pi r_*}{m_* c^2} = \frac{e^2 / 2\pi r_e}{m_e c^2}. \quad (19)$$

The Schwinger ratios in (19) can be directly related to the structure of the PV state: the $2\pi r_*$ is the “circumference” of the “sphere” of “radius” r_* that contains the Planck particle core. This strong correspondence suggests a physical relationship between the QED and PV theories.

V. PROTON G-FACTOR

The electron is thought to be a true point particle [6, p.82] because it contains no internal structure, as does the proton [9]. In the present context, however, it is appropriate to associate the “size” of the electron and proton with their Compton radii, where the corresponding proton structure constant is defined here as

$$m_p = \frac{r_e}{r_p} m_e \quad \rightarrow \quad \left(\frac{r_e}{r_p} \right) = \frac{m_p}{m_e} \approx 1836. \quad (20)$$

This relatedness suggests that the proton g-factor change be estimated from the electron change,

$$\frac{g}{2} - 1 = \frac{\alpha}{2\pi} \frac{r_e}{r_p} = 0.001162 \frac{r_e}{r_p} = 2.13 \quad (21)$$

where the experimental g-factor is [6, p.82]

$$\left(\frac{g}{2} - 1 \right)_{exp} = 1.79. \quad (22)$$

The agreement between (21) and (22) is remarkable, considering the large magnitude of r_e / r_p . It remains to be seen, however, whether or not (21) leads to something more substantial.

VI. SUMMARY AND COMMENTS

It comes as a surprise that the charge associated with the Dirac equation is the coupling charge e_* , rather than the well known electron-proton charge magnitude e . That result is due to the collection of Planck particle cores ($\pm e_*$, m_*) that pervade the PV state. If there were no such pervasion, there would be no photon scattering taking place off those cores; and no need for the coupling charge in the Dirac equation, or

the radiative corrections from the QED theory. In that case, the Dirac equation would provide a self-sufficient description of the electron and proton particles, incorporating the squared charge e^2 .

It is noted that the electron or proton core velocities v appear nowhere in the Dirac equations (8) or (9). This is due to the fact that the equations are Lorentz invariant, where the velocities are tacitly assumed to lie anywhere within the half-open interval $0 \leq v < c$ as the cores propagate within their Lorentz reference frames.

As an aside, the radiative correction α even shows up in the Bohr hydrogen atom calculations [10], where the electron and proton are considered to be point particles. For example, the expression

$$nv_n = \alpha c \quad (23)$$

for the velocity v_n of the electron in its n th circular orbit (around the electron-proton center of mass) clearly displays the α correction. Without the correction, the unrealistic result $nv_n = c$ is obtained for the electron velocity.

We close with the following quote [11, p.129] from Feynman: “There is a most profound and beautiful question associated with the coupling constant, e —the amplitude for a real electron to emit or absorb a real photon. It is a simple number that has been experimentally determined to be close to -0.8542455 . (My physicist friends won’t recognize this number, because they like to remember it as the inverse of its square: about 137.03597 with an uncertainty of about 2 in the last decimal place. It [the fine structure constant] has been a mystery ever since it was first discovered more than fifty years ago, and all good theoretical physicists put this number up on their wall and worry about it.)” Calculations in the present paper appear to have removed that mystery.

APPENDIX A THE γ AND β MATRICES

The 4x4 γ , β , and α_i matrices used in the Dirac theory are defined here: where [6, p.91]

$$\gamma^0 \equiv \beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \quad (A1)$$

and ($j = 1, 2, 3$)

$$\gamma^j \equiv \beta \alpha_j = \begin{pmatrix} 0 & \sigma_j \\ -\sigma_j & 0 \end{pmatrix} \quad (A2)$$

and where I is the 2x2 unit matrix and

$$\alpha_j = \begin{pmatrix} 0 & \sigma_j \\ \sigma_j & 0 \end{pmatrix} \quad (A3)$$

where the σ_j are the 2x2 Pauli spin matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad (A4)$$

and $\alpha = (\alpha_1, \alpha_2, \alpha_3)$. The zeros in (A1)–(A3) and (A5) are 2x2 null matrices.

The mc^2 in (8) and (9) represents the 4x4 matrix

$$mc^2 \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix} \quad (A5)$$

and ψ is the 4x1 spinor matrix.

The zero on the right side of (9) represents the 4x4 null matrix and the zeros in (10) represent 2x2 null matrices. The S_j and σ_j in (10) are 2x2 matrices; so their parentheses represent 4x4 matrices.

The coordinates x^μ are

$$x^\mu = (x^0, x^1, x^2, x^3) \quad (A6)$$

where $x^0 \equiv ct$.

REFERENCES

- [1] P. Davies, *Superforce: the Search for a Grand Unified Theory of Nature*. Simon and Schuster, Inc., New York, 1984.
- [2] W.C. Daywitt, A Model for Davies’ Universal Superforce. *Galilean Electrodynamics*, vol.5, (2006) (Sept./Oct.), 83. See also www.planckvacuumDOTcom.
- [3] W.C. Daywitt, The Trouble with the Equations of Modern Fundamental Physics. *American Journal of Modern Physics*. Special Issue: Physics without Higgs and without Supersymmetry, v.5, (2016), no.1-1, 22.
- [4] B.W. Carroll, Ostlie D.A.: *An Introduction to Modern Astrophysics*. Addison-Wesley, San Francisco—Toronto, (2007).
- [5] W.C. Daywitt, The Planck vacuum. *Progress in Physics*, v.1, 20, 2009.
- [6] D.M. Gingrich, *Practical Quantum Electrodynamics*. CRC, The Taylor & Francis Group, Boca Raton, London, New York, 2006.
- [7] R.B. Leighton, *Principles of Modern Physics*. McGraw-Hill Book Co., New York, Toronto, London, 1959.
- [8] Julian Schwinger, *On Quantum-Electrodynamics and the Magnetic Moment of the Electron*. *Physical Review*, 1948, Vol.73, p.416.
- [9] W.C. Daywitt, The Structured Proton and the Structureless Electron as Viewed in the Planck Vacuum Theory. *Progress in Physics*, v.11, Issue 2 (April), 117, 2015.
- [10] W.C. Daywitt, The Bohr Hydrogen Atom as Viewed in the Planck Vacuum Theory. *Galilean Electrodynamics*, 2019. See also www.planckvacuumDOTcom.
- [11] R.P. Feynman, *QED: the strange theory of light and matter*. Princeton University Press, Princeton and Oxford, UK, 1985.