

# The Dirac Electron and Its Propagator as Viewed in the Planck Vacuum Theory

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This paper examines the covariant Dirac equation and its associated quantum-electrodynamic propagator from the perspective of the Planck vacuum (PV) theory. Calculations reveal: that the PV state is a bifurcated state whose two branches provide the electrons and positrons that, under certain conditions, can be scattered from the PV into free space; that the degenerate collection of Planck-particle cores (that pervade the invisible, negative-energy vacuum state) is responsible for the scattering that takes place in the Huygens principle and the propagator theory; and that the two-term coupling force the electron core exerts on the PV state vanishes at the electron Compton radius, preventing the electron core (and its consequent Dirac electron) from being tethered by the coupling force to the vacuum state, assuring that the electron propagates freely in free space. The paper represents a relativistic addendum to an earlier paper [1] concerning the Schrödinger electron.

## 1 Introduction

Charge conjugation [2] in the PV theory implies that the invisible vacuum state must be a bifurcated state — bifurcation meaning that at each point in free space there exists a vacuum subspace consisting of the charge doublet  $(\pm e_*)^2$  that defines the two vacuum branches

$$e_*^2 = (-e_*)(-e_*) \quad \text{and} \quad e_*^2 = (+e_*)(+e_*). \quad (1)$$

The first charge in each branch belongs to the electron or positron and the second charge to the corresponding branch of the subspace. For example, if the first charge  $(-e_*)$  in the negative branch on the left belongs to the electron, then the first charge  $(+e_*)$  in the positive branch at the right belongs to the positron. In other words, in the PV theory charge conjugation simply switches back and forth between the two branches. The equivalence of the two branches can be seen in the Dirac equation

$$i\hbar \left( \frac{\partial}{\partial t} + \boldsymbol{\alpha} \cdot \nabla \right) \psi = mc^2 \beta \psi \quad (2)$$

or, using  $c\hbar = e_*^2$ ,

$$\left[ i(-e_*)(-e_*) \left( \frac{\partial}{\partial t} + \boldsymbol{\alpha} \cdot \nabla \right) - mc^2 \beta \right] \psi = 0 \quad (3)$$

where the negative branch, the electron branch, is used. The Dirac equation (2) applies to both branches; i.e. the equation works for both the electron and positron. A similar statement can also be made for the equations in (5).

The theoretical foundation [3, 4, 5] of the PV theory rests upon the unification of the Einstein, Newton, and Coulomb superforces:

$$\frac{c^4}{G} \left( = \frac{m_* c^2}{r_*} \right) = \frac{m_*^2 G}{r_*^2} = \frac{e_*^2}{r_*^2} \quad (4)$$

where the ratio  $c^4/G$  is the curvature superforce that appears in the Einstein field equations.  $G$  is Newton's gravitational

constant,  $c$  is the speed of light,  $m_*$  and  $r_*$  are the Planck mass and length respectively [6, p.1234], and  $e_*$  is the massless bare charge. The fine structure constant is given by the ratio  $\alpha = e^2/e_*^2$ , where  $(-e)$  is the observed electronic charge.

The two particle/PV coupling forces

$$F_c(r) = \frac{e_*^2}{r^2} - \frac{mc^2}{r} \quad \text{and} \quad F_*(r) = \frac{e_*^2}{r^2} - \frac{m_*c^2}{r} \quad (5)$$

the electron core  $(-e_*, m)$  and the Planck-particle core  $(-e_*, m_*)$  exert on the PV state, along with their coupling constants

$$F_c(r_c) = 0 \quad \text{and} \quad F_*(r_*) = 0 \quad (6)$$

and the resulting Compton radii

$$r_c = \frac{e_*^2}{mc^2} \quad \text{and} \quad r_* = \frac{e_*^2}{m_*c^2} \quad (7)$$

lead to the important string of Compton relations

$$r_c mc^2 = r_* m_* c^2 = e_*^2 \quad (= c\hbar) \quad (8)$$

for the electron and Planck-particle cores, where  $\hbar$  is the reduced Planck constant. The electron and Planck particle masses are  $m$  and  $m_*$  respectively. The vanishing of  $F_c(r_c)$  in (6) frees the electron from being tethered to the vacuum state, insuring that the electron propagating in free space behaves as a free particle.

The Planck constant is a secondary constant whose structure can take different forms; e.g.

$$\hbar [\text{erg sec}] = r_c mc = r_* m_* c = \left( \frac{e_*^2}{r_*} \right) t_* = m_* c^2 t_* \quad (9)$$

that are employed throughout the following text, where  $t_*$  ( $= r_*/c$ ) is the Planck time [6, p.1233]. The products to the right of  $\hbar$  relate the electron mass  $m$  and Compton radius  $r_c$  to the vacuum parameters  $r_*$ ,  $m_*$ ,  $t_*$ , and  $e_*^2$ .

Furthermore, the energy and momentum operators expressed as

$$\widehat{E} = i\hbar \frac{\partial}{\partial t} = i(m_*c^2)t_* \frac{\partial}{\partial t} = i(m_*c^2)r_* \frac{\partial}{c\partial t} \quad (10)$$

and

$$c\widehat{\mathbf{p}} = -i\hbar\nabla = -i(m_*c^2)r_*\nabla = -i(mc^2)r_c\nabla \quad (11)$$

will be used freely in what follows.

Section 2 examines the covariant Dirac equation and the covariant Dirac equation with the electromagnetic interaction included. Results show that the two equations can be totally normalized by the vacuum parameters  $r_*$  and  $m_*c^2$  from (8).

Section 3 looks at the relativistic Dirac propagator that provides the foundation for the scattering in the Huygens-principle and the propagator formalisms. The propagator equation is normalized by the vacuum parameters  $r_*$  and  $m_*c$  from (9).

Section 4 traces the scatterings of the Huygens principle and the propagator theory to the pervaded vacuum space, and indicates how electron-positron pair creation is related to PV charge conjugation.

## 2 Dirac equation

The manifestly covariant form of the Dirac equation [7, p.90] is

$$\left(i\hbar\gamma^\mu \frac{\partial}{\partial x^\mu}\right)\psi - mc\psi = 0 \quad (12)$$

which, using (9), can be expressed as

$$\left(i\gamma^\mu \frac{r_*\partial}{\partial x^\mu}\right)\psi - \frac{mc}{m_*c}\psi = 0 \quad (13)$$

with

$$\frac{\partial}{\partial x^\mu} \equiv \left(\frac{\partial}{c\partial t}, \nabla\right) \quad (14)$$

where  $\psi$  is the 4x1 Dirac spinor, [ $\mu = 0, 1, 2, 3$ ], and  $\nabla$  is the normal 3-dimensional gradient operator. See Appendix A for the definition of the  $\gamma^\mu$  matrices. The summation convention over the two  $\mu$ s in the first terms of (12) and (13) is understood.

The Dirac equation with the electromagnetic interaction included is [7, eqn.5.249]

$$\left[i\hbar\gamma^\mu \frac{\partial}{\partial x^\mu} \pm \frac{e\gamma^\mu A_\mu}{c}\right]\psi - mc\psi = 0 \quad (15)$$

which, using (9), can be reduced to

$$\left[i\gamma^\mu \frac{r_*\partial}{\partial x^\mu} \pm \frac{e\gamma^\mu A_\mu}{m_*c^2}\right]\psi - \frac{mc}{m_*c}\psi = 0 \quad (16)$$

where the minimal-substitution ratio [7, p.90]

$$\pm \frac{e\gamma^\mu A_\mu}{c} \quad (17)$$

represents the relativistic electromagnetic interaction of the charge ( $\mp e$ ) with the 4-potential  $A_\mu$ .

## 3 Dirac propagator

The relativistic Dirac propagator  $S_F(x', x; A)$  is defined to satisfy the Green-function equation [7, eqn.6.91]

$$\left[\gamma_\mu \left(i\hbar \frac{\partial}{\partial x'_\mu} - \frac{eA^\mu(x')}{c}\right) - mc\right]_{\alpha\lambda} S_{F,\beta}(x', x; A) = \delta_{\alpha\beta}\delta^4(x' - x) \quad (18)$$

which reduces to

$$\left[\gamma_\mu \left(i \frac{r_*\partial}{\partial x'_\mu} - \frac{\alpha^{1/2}e_*A^\mu(x')}{m_*c^2}\right) - \frac{mc}{m_*c}\right]_{\alpha\lambda} S_{F,\beta}(x', x; A) = \delta_{\alpha\beta} \frac{\delta^4(x' - x)}{m_*c} \quad (19)$$

where  $e = \alpha^{1/2}e_*$  is used in the reduction and  $\delta_{\alpha\beta}$  is the Kronecker delta. The bracket on the left is dimensionless and the  $\delta^4$  on the right has the units of “1/spacetime-volume”. Thus  $S_F$  in (19) has the units “1/mc·spacetime-volume”.

## 4 Conclusions and comments

The product  $m_*c^2$  in (8) is the upper limit to elementary-particle mass-energy and  $r_*$  is the lower limit to the particle Compton radius. With this in mind, and the fact the normalizers in equations (13), (16), and (19) are  $m_*c$  and  $r_*$ , it is assumed in the PV theory that the Planck-particle cores ( $\pm e_*, m_*$ ) associated with the two branches in (1) that pervade the PV state are the scatterers that provide the scattering for the Huygens-principle and the propagator formalisms. For example, in (13)  $r_*$  normalizes the four spacetime gradients  $\partial/\partial x^\mu$  and  $m_*c$  normalizes the electron product  $mc$ .

Finally, the charge ambiguity in (2) due to (1) allows for the creation of an electron-positron pair [7, fig.6.6],

$$\left[i(-e_*)(-e_*)\left(\frac{\partial}{c\partial t} + \boldsymbol{\alpha} \cdot \nabla\right) - mc^2\beta\right]\psi = 0 \oplus \left[i(+e_*)(+e_*)\left(\frac{\partial}{c\partial t} + \boldsymbol{\alpha} \cdot \nabla\right) - mc^2\beta\right]\psi = 0, \quad (20)$$

where the first and second equations are related respectively to the electron and positron branches in (1).

## Appendix A: The $\gamma$ and $\beta$ matrices

The 4x4  $\gamma, \beta$ , and  $\alpha_i$  matrices used in the Dirac and propagator theories are defined here: where [7, p.75]

$$\gamma^0 \equiv \beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \quad (A1)$$

and

$$\gamma^i \equiv \beta\alpha_i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix} \quad (A2)$$

and where  $I$  is the  $2 \times 2$  unit matrix and

$$\alpha_i = \begin{pmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{pmatrix} \quad (\text{A3})$$

where the  $\sigma_i$  are the  $2 \times 2$  Pauli matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad (\text{A4})$$

and  $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, \alpha_3)$ .

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