The Bohr Hydrogen Atom as Viewed in the Planck Vacuum Theory

William C. Daywitt

National Institute for Standards and Technology (retired), Boulder, Colorado. E-mail: wcdaywitt@me.com

This paper examines the Bohr hydrogen-atom model from the perspective of the Planck vacuum (PV) theory. The nonrelativistic calculations significantly expand the Bohr concept concerning the quantized angular momentum, the hydrogen energy levels E_n, the orbit radii r_n, the ratios E_n/r_n, and the hydrogen Rydberg constant R_H. Calculations show: that the Bohr quantization of the angular momentum is directly related to the electron/PV coupling force; and that the ratios E_n/r_n are proportional to the n-ratio from the Schwarzschild line element for the Einstein field equations.

[Reference: W.C. Daywitt, The Bohr Hydrogen Atom as Viewed in the Planck Vacuum Theory. To be published in Galilean Electrodynamics 2018.]

1 Introduction

The theoretical foundation [1] [2] [3] of the PV theory rests upon the unification of the Einstein, Newton, and Coulomb superforces:

$$\frac{c^4}{G}\left(=\frac{m_*c^2}{r_*}\right)=\frac{m_*^2G}{r_*^2}=\frac{e_*^2}{r_*^2} \tag{1}$$

where the ratio c^4/G is the curvature superforce that appears in the Einstein field equations. G is Newton's gravitational constant, c is the speed of light, m_* and r_* are the Planck mass and length respectively [4, p.1234], and e_* is the massless bare charge. The fine structure constant is given by the ratio $\alpha=e^2/e_*^2$, where $(-e)$ is the observed electronic charge.

The two particle/PV coupling forces

$$F_c(r)=\frac{e_*^2}{r^2}-\frac{mc^2}{r} \quad \text{and} \quad F_*(r)=\frac{e_*^2}{r^2}-\frac{m_*c^2}{r} \tag{2}$$

the electron core $(-e_*,m)$ and the Planck particle core $(-e_*,m_*)$ exert on the invisible PV state; along with their coupling constants

$$F_c(r_c)=0 \quad \text{and} \quad F_*(r_*)=0 \tag{3}$$

and the resulting Compton radii

$$r_c=\frac{e_*^2}{mc^2} \quad \text{and} \quad r_*=\frac{e_*^2}{m_*c^2} \tag{4}$$

lead to the important string of Compton relations

$$r_cmc^2=r_*m_*c^2=e_*^2 \quad (=c\hbar) \tag{5}$$

for the electron and Planck-particle cores, where $\hbar$ is the reduced Planck constant. To reiterate, the equations in (2) represent the forces the free electron and Planck-particle cores exert on the invisible PV space, a space that is itself pervaded by a degenerate collection of Planck-particle cores [5].

The Lorentz invariance of the coupling constants in (3) leads to the energy and momentum operators of the quantum theory [5].

The various PV parameters in the previous paragraphs are used in Section 3 to unravel the Bohr-atom results in Section 2 from the single Planck constant $\hbar$ contained therein, leading to a deeper understanding of the Bohr atom defined below.

2 Bohr Atom

The Bohr atom [6] [7, p.73] consists of a massive point-electron $(-e,m)=(-\alpha^{1/2}e_*,m)$ and a massive point-proton $(e,M)=(\alpha^{1/2}e_*,M)$ circling their common center of mass. Their masses and circling radii are related by the equation $mr=MR$, where r and R in this equation represent these radii. The r here IS NOT the same as the coordinate radius r in equations (2).

The quantized angular momentum assumed by Bohr is given by [7, p.73]

$$(mr^2+MR^2)\omega=mr^2\omega(1+m/M)=n\hbar \tag{6}$$

in terms of the Planck constant $\hbar$. The centripetal force driving the electron toward the center of mass is given by

$$m\omega^2r=\frac{e^2}{(r+R)^2}=\frac{e^2}{r^2(1+m/M)^2} \tag{7}$$

and the total energy $T+V$ by (using (7))

$$E_n=\frac{mr^2\omega^2}{2}+\frac{MR^2\omega^2}{2}-\frac{e^2}{r+R}$$

$$=-\frac{mr^2\omega^2}{2}(1+m/M) \tag{8}$$

where ω is the angular frequency of the electron and proton about their common center of mass.

Each of the equations (6)–(8) may be solved for $\omega^2 r^4$:

$$\omega^2 r^4 = \frac{n^2\hbar^2}{m^2(1+m/M)^2} = A$$

$$= \frac{e^2 r}{m(1+m/M)^2} = rB$$

$$= -\frac{2E_n r^2}{m(1+m/M)} = r^2(-E_n)C \,. \tag{9}$$

Eliminating r from the right sides of the above equalities then leads to the energy spectrum $(n = 1, 2, 3, \ldots)$

$$E_n = \frac{B^2}{AC} = -\frac{me^4}{2n^2\hbar^2(1+m/M)} \tag{10}$$

and the electron-orbit radii

$$r_n = \frac{A}{B} = \frac{n^2\hbar^2}{me^2} \,. \tag{11}$$

The wavenumber for the radiation between the energy states n' and n is given by

$$\frac{\nu}{c} = \frac{E_n - E_{n'}}{2\pi c\hbar} = R_H\left(\frac{1}{n'^2} - \frac{1}{n^2}\right) \tag{12}$$

where the Rydberg constant is

$$R_H = \frac{me^4}{4\pi c\hbar^3(1+m/M)} \,. \tag{13}$$

These results all depend upon the single constant $\hbar$; so their fundamental physical meaning is obscured. The next section remedies this shortcoming.

3 Planck Vacuum Perspective

This section replaces the single constant $\hbar$ of the previous section by the constants from the PV theory given in the Introduction. It is left for the reader to fill-in the simple steps. The new results due to the PV theory pre-multiply the bracketed quantities in the following equations.

The new angular momentum from (6) becomes

$$(mr^2 + MR^2)\omega = mr^2\omega(1+m/M)$$

$$= [n\hbar] = \frac{e_*^2}{c}[n] = r_c mc[n] \,. \tag{14}$$

The squared charge $e_*^2 = (-e_*)(-e_*)$ in the first equation of (2) represents the numerator of the Coulomb force the electron (the first charge) exerts on the separate Planck particles (the second charge) of the PV state. As such, its appearance in (14) is a manifestation of that coupling force. The $r_c mc$ in (14) comes from the Compton relations in (4). Thus, from the first equation in (3), the quantized angular momentum is restricted to those values $(r_c mc)$ where the electron coupling force $F_c(r_c)$ acting on the vacuum state vanishes, and there are a denumerable infinity $(n = 1, 2, 3, \ldots)$ of such values. For $r > r_c$ the electron force in (2) is attractive and for $r < r_c$ it is repulsive.

The new energy levels are given by

$$E_n = -mc^2\alpha^2\left[\frac{1}{2n^2(1+m/M)}\right] \tag{15}$$

where mc^2 is the mass energy of the electron and α is the fine structure constant.

The new Bohr electron orbits are

$$r_n = \frac{r_c}{\alpha}[n^2] = \frac{e_*^2}{\alpha mc^2}[n^2] \tag{16}$$

where $r_c(= e_*^2/mc^2)$ is the electron Compton radius. This result yields the correct value for r_1 with e_* in cgs-esu units [7, p.722].

The energy/orbit ratios are

$$\frac{E_n}{r_n} = -\frac{mc^2}{r_c}\alpha^3\left[\frac{1}{2n^4(1+m/M)}\right] \tag{17}$$

$$= -\frac{m_*c^2}{r_*}n_{rc}\alpha^3\left[\frac{1}{2n^4(1+m/M)}\right] \tag{18}$$

where

$$n_{rc} \equiv \frac{mc^2/r_c}{m_*c^2/r_*} \tag{19}$$

is the n-ratio that appears in the Schwarzschild line element for the Einstein field equations [8].

The new hydrogen-atom Rydberg constant is given by

$$R_H = \frac{\alpha^2}{r_c}\left[\frac{1}{4\pi(1+m/M)}\right] \,. \tag{20}$$

This result yields the correct value for the Rydberg constant in cgs-esu units [7, p.723].

4 Conclusions and Comments

It is noted that all of the pre-multipliers in equations (14)–(18) and (20) are PV parameters. Thus the PV theory represents a significant addition to the previous Bohr-atom analysis.

Since the angular momentum has been quantized, the classical variables that are part of the analysis take on new meanings. For example, the angular frequency becomes [for convenience $(1 + m/M) = 1$]

$$\omega = \frac{r_c mc[n]}{mr^2} = \frac{r_c c[n]}{r_c^2[n^4]/\alpha^2} = \alpha\frac{\alpha c}{r_c}\left[\frac{1}{n^3}\right] \tag{21}$$

where, from (23), αc is the velocity of the electron in the first $(n = 1)$ Bohr orbit around the center of mass.

Using (21) for ω, the centripetal acceleration becomes

$$\omega^2 r = \alpha^4 \frac{c^2}{r_c^2} \left[\frac{1}{n^6} \right] \frac{r_c}{\alpha} [n^2] = \alpha \frac{\alpha^2 c^2}{r_c} \left[\frac{1}{n^4} \right] \qquad (22)$$

and the orbit velocity of the electron becomes

$$v = \omega r = \alpha^2 \frac{c}{r_c} \left[\frac{1}{n^3} \right] \frac{r_c}{\alpha} [n^2] = \alpha c \left[\frac{1}{n} \right] . \qquad (23)$$

References

[1] P. Davies, Superforce: the Search for a Grand Unified Theory of Nature, (Simon and Schuster, Inc., New York, 1984).

[2] W.C. Daywitt, "A Model for Davies' Universal Superforce", Galilean Electrodynamics, vol.5, (Sept./Oct.), 83, (2006). See also www.planckvacuumDOTcom.

[3] W.C. Daywitt, "The Trouble with the Equations of Modern Fundamental Physics", American Journal of Modern Physics, Special Issue: Physics without Higgs and without Supersymmetry. vol.5, no.1-1, 22, (2016).

[4] B.W. Carroll, D.A. Ostlie, An Introduction to Modern Astrophysics, (Addison-Wesley, San Francisco, Boston, New York, Cape Town, Hong Kong, London, Madrid, Mexico City, Montreal, Munich, Paris, Singapore, Sydney, Tokyo, Toronto, 2007).

[5] W.C. Daywitt, The Planck Vacuum. Galilean Electrodynamics, vol.4, 72, 2010.

[6] N. Bohr, "On the Constitution of Atoms and Molecules", Philos. Mag., 26, 1, 1913.

[7] R.B. Leighton, Principles of Modern Physics, (McGraw-Hill Book Co., New York, Toronto, London, 1959). The present paper uses cgs-esu units; so the use of $e^2/4\pi\epsilon_0$ in this reference should be replaced by just e^2 for the present analysis.

[8] W.C, Daywitt, "The Planck Vacuum and the Schwarzschild Metrics", Progress in Physics, vol.3, 30, (2009).