The Apparent Lack of Lorentz Invariance in Zero-Point Fields with Truncated Spectra

William C. Daywitt

National Institute for Standards and Technology (retired), Boulder, Colorado, USA

E-mail: wcdaywitt@earthlink.net

The integrals that describe the expectation values of the zero-point quantum-field-theoretic vacuum state are semi-infinite, as are the integrals for the stochastic electrodynamic vacuum. The unbounded upper limit to these integrals leads in turn to infinite energy densities and renormalization masses. A number of models have been put forward to truncate the integrals so that these densities and masses are finite. Unfortunately the truncation apparently destroys the Lorentz invariance of the integrals. This note argues that the integrals are naturally truncated by the graininess of the negative-energy Planck vacuum state from which the zero-point vacuum arises, and are thus automatically Lorentz invariant.

1 Introduction

Sakharov [1] hypothesized that Newton's gravitational constant is inversely proportional to a truncated integral over the momenta of the virtual particles in the quantum vacuum [2] (QV), and that the cutoff wavenumber "…determines the mass of the heaviest particles existing in nature…" according to a suggestion by M. A. Markov. Inverting the Markov suggestion, the Planck vacuum (PV) model [3, 4] assumes that these "heaviest particles" are the Planck particles (PPs) constituting the degenerate negative-energy PV state, and that it is the separation between these PPs that leads to the cutoff wavenumber. Puthoff [5, 4] furthers the Sakharov argument by calculating the cutoff wavenumber to be

$$k_{c*} = \left(\frac{\pi c^3}{\hbar G}\right)^{1/2} \quad \left[= \frac{\pi^{1/2}}{r_*}\right], \tag{1}$$

where G is Newton's gravitational constant and r_* is the Planck length. The ratio in the bracket is derived by substituting the constants $\hbar = e_*^2/c$, $G = e_*^2/m_*^2$, and the Compton relation $r_* m_* c^2 = e_*^2$ from the PV model, where m_* is the Planck mass and e_* is the bare (true) charge common to the charged elementary particles.

It is accepted knowledge that the truncation of the vacuum integrals destroys their Lorentz invariance. For example, a stochastic electrodynamic version of the zero-point (ZP) electric field can be expressed as [5]

$$\mathbf{E}_{zp}(\mathbf{r},t) = \mathrm{Re} \sum_{\sigma=1}^{2} \int d\Omega_k \int_0^{k_{c*}} dk\, k^2\, \mathbf{e}_\sigma(\mathbf{k})\, A_k \times$$
$$\times \exp\left[\, i\,(\mathbf{k}\cdot\mathbf{r} - \omega t + \Theta_\sigma(\mathbf{k}))\,\right], \tag{2}$$

where the cutoff wavenumber k_{c*} apparently destroys the Lorentz invariance of the field. The accepted Lorentz-invariant version of (2) replaces k_{c*} by ∞. By giving the cutoff wavenumber an interpretation different from a momentum wavenumber, however, this note argues that (2) is Lorentz invariant as it stands. The next section presents this argument.

The virtual-particle field consists of virtual photons and massive virtual-particle pairs, the collection being the QV. It is assumed that the structure of the PV and the ZP agitation of its PPs are responsible for the structure of the virtual-particle field, the corresponding average of the photon field being the ZP electric field in (2). While the negative-energy PV is assumed to be invisible (not directly observable), it offspring the QV appears in free space and interacts with the free particles therein. The argument in the next section assumes this perspective.

2 Cutoff wavenumber

The set of orthogonal modes associated with a continuous medium contains an infinite number of eigenfunctions. If the medium is quasi-continuous like the PV, however, the number is finite. Using this fact, the development of the ZP electric field is reviewed below to show that the cutoff wavenumber is associated with the number of PPs per unit volume in the PV and is not fundamentally a momentum wavenumber for the QV fields. Thus being associated with the PP density, the cutoff wavenumber is not dependent upon the free-space Lorentz frames observing the QV.

The ZP electric field can be expressed as [6, p.73]

$$\mathbf{E}_{\mathrm{zp}}(\mathbf{r},t) = \left(\frac{8\pi^3}{V}\right)^{1/2} \times$$
$$\times\ \mathrm{Re} \sum_{\sigma} \sum_{\mathbf{n}} \mathbf{e}_{\mathbf{k},\sigma} A_k \exp\left[\, i\,(\mathbf{k}\cdot\mathbf{r} - \omega t + \Theta_{\mathbf{k},n})\right], \tag{3}$$

where the first sum is over the two polarizations of the field, $k = |\mathbf{k}|$, $V = L^3$ is the box-normalization volume, $\mathbf{e}_{\mathbf{k},\sigma}$ is the polarization vector,

$$\pi^2 A_k^2 = \frac{\hbar\omega}{2} = \frac{e_*^2 k}{2} \tag{4}$$

yields the amplitude factor A_k which is proportional to the bare charge e_* of the PPs in the PV, and Θ_σ is the random phase that gives $\mathbf{E}_{\mathrm{zp}}$ its stochastic character. The two ratios in (4) are the ZP energy of the individual field modes. The field satisfies the periodicity condition

$$\mathbf{E}_{\mathrm{zp}}(x+L,\ y+L,\ z+L,t) = \mathbf{E}_{\mathrm{zp}}(x,y,z,t) \tag{5}$$

or equivalently

$$\mathbf{k} = (k_x,k_y,k_z) = (2\pi/L)(n_x,n_y,n_z) = (2\pi/L)\,\mathbf{n}\,, \tag{6}$$

where $k = (2\pi/L)\,n$, and where ordinarily the n_i can assume any positive or negative integer and zero.

An unbounded mode index n_i in (6) leads to the infinite energy densities and renormalization masses that plague both the quantum field theory and the stochastic electrodynamic theory. However, if the normal mode functions of the ZP field are assumed to be waves supported by the collection of PPs within the PV [4], then the number of modes n_i along the side of the box of length L is bounded and obeys the inequality $|n_i| \leqslant (L/2\pi)\,k_{c*} = L/2\sqrt{\pi}\,r_*$. So it is the "graininess" ($r_* \neq 0$) associated with the minimum separation r_* of the PPs that leads to a bounded k_i and n_i for (6), and which is thus responsible for finite energy densities and renormalization masses [4]. Unfortunately this truncation of the second sum in (3) leads to apparently non-Lorentz-invariant integrals for the "continuum" version of that equation developed below.

Using the replacement [6, p.76]

$$\sum_\sigma \sum_{\mathbf{n}} f(\mathbf{k}_{\mathbf{n}}, \mathbf{e}_{\mathbf{n},\sigma})\, a_{\mathbf{n},\sigma} \longrightarrow \tag{7}$$

$$\longrightarrow \left(\frac{V}{8\pi^3}\right)^{1/2} \sum_\sigma \int d^3k\, f(\mathbf{k}, \mathbf{e}_\sigma(\mathbf{k}))\, a_\sigma(\mathbf{k}) \tag{8}$$

in (3) and truncating the field densities at $k_{c*} = \sqrt{\pi}/r_*$ leads to [5, 4]

$$\begin{aligned} \mathbf{E}_{\mathrm{zp}}(\mathbf{r},t) &= \mathrm{Re} \sum_\sigma \int d^3k\, \mathbf{e}_\sigma(\mathbf{k})\, A_k \times \\ &\times \exp\left[\, i\,(\mathbf{k}\cdot\mathbf{r} - \omega t + \Theta_\sigma(\mathbf{k}))\,\right] = \\ &= \mathrm{Re} \sum_\sigma \int d\Omega_k \int_0^{k_{c*}} dk\, k^2\, \mathbf{e}_\sigma(\mathbf{k})\, A_k \times \\ &\times \exp\left[\, i\,(\mathbf{k}\cdot\mathbf{r} - \omega t + \Theta_\sigma(\mathbf{k}))\,\right], \end{aligned} \tag{9}$$

where $d\Omega_k$ is the k-space solid-angle differential. As shown below in (10) and(11) this cutoff wavenumber k_{c*} is fundamentally related to the number of PPs per unit volume constituting the PV.

The ZP electromagnetic energy density of the QV calculated from (8) is

$$\frac{\langle \mathbf{E}^2_{\mathrm{zp}} \rangle}{4\pi} = \int_0^{k_{c*}} \frac{e_*^2 k}{2} \cdot \frac{k^2\, dk}{\pi^2}\,, \tag{10}$$

where the first ratio under the integral sign is the ZP energy of the individual modes. The second ratio is the number of modes per unit volume between k and $k + dk$; so the number of modes in that range is $k^2 V\, dk/\pi^2$. If the total number of PP oscillators (with three degrees of freedom each) in the volume V is N, then the total number of modes in V is [7]

$$\int_0^{k_{c*}} \frac{k^2 V\, dk}{\pi^2} = 3N\,, \tag{11}$$

which provides an estimate for N/V. Integrating (10) gives

$$\frac{N}{V} = \frac{k^3_{c*}}{9\pi^2} \quad \left[= (9^{1/3}\pi^{1/6} r_*)^{-3} \approx \frac{1}{(2.5\,r_*)^3}\right] \tag{12}$$

for the number of PPs per unit volume. The equation outside the brackets shows that k_{c*} is proportional to the cube root of this PP density. The ratio in the bracket shows that the average separation of the PPs is approximately 2.5 times their Compton radii r_*, a very reasonable result considering the roughness of the calculations.

From (11) the previous paragraph shows that the cutoff wavenumber k_{c*} in (8) and (9) is associated with the mode counting in (10) taking place within the invisible PV. Since the number of these PV modes is not influenced by the free-space Lorentz frame observing the QV, the k_{c*} in (8) and (9) must be independent of the Lorentz frame. Thus (8) and (9) are Lorentz invariant as they stand since k_{c*} is frame independent and the integrands are already Lorentz invariant [8]. That is, when viewed from different Lorentz frames, the wavenumber k_{c*} remains the same; so the integrals are Lorentz invariant.

3 Review and comments

From the beginning of the ZP theory the medium upon which calculations are based is the free-space continuum with its unbounded mode density. So if the spectral density is truncated, the ZP fields naturally lose their Lorentz-invariant character because the truncation and the Lorentz viewing frames exist in the same space. This contrasts with the development in the preceding section where the truncation takes place in the invisible PV while the viewing is in the free space containing the QV.

One way of truncating in free space without losing Lorentz invariance [9, 10] is to assume that the so-called elementary particles are constructed from small sub-particles called partons, so that the components of the parton driving-field $\mathbf{E}_{\mathrm{zp}}$ with wavelengths smaller than the parton size ($\sim r_*$) are ineffective in producing translational motion of the parton as a whole, effectively truncating the integral expressions at or near the Planck frequency c/r_*. The parton mass turns out to be

$$m_0 = \frac{2}{3}\left(\frac{m_*^2}{m}\right) = \frac{2}{3}\left(\frac{r_c}{r_*}\right) m_* \sim 10^{20}\, m_* \tag{13}$$

where m_* is the Planck mass, m is the particle mass, and r_c is the particle Compton radius. The parenthetical ratio in the second expression is roughly 10^{20} for the observed elementary particles; i.e., for the observed particles, the parton mass is about twenty orders of magnitude greater than the Planck mass.

It is difficult to explain the inordinately large ($10^{20} m_*$) parton mass in (12) that is due to the equation of motion

$$m_0 \ddot{\mathbf{r}} = e_* \mathbf{E}_{\rm zp} \tag{14}$$

at the core of the Abraham-Lorentz-Dirac equation used in [9], where $\ddot{\mathbf{r}}$ is the acceleration of the mass about its average position at $\langle \mathbf{r} \rangle = 0$. Equation (13) is easily transformed into the equation of motion

$$e_* \ddot{\mathbf{r}} = \frac{3c^3 \Gamma}{2} \mathbf{E}_{\rm zp} \tag{15}$$

for the charge e_*, where $\ddot{\mathbf{r}}$ is the charge acceleration. If the time constant Γ is treated as a constant to be determined from experiment [5, 4], then solving (14) leads to

$$\Gamma = \left(\frac{r_*}{r_c}\right) \frac{r_*}{c} \sim 10^{-20} \, \frac{r_*}{c} \,, \tag{16}$$

where r_*/c is the Planck time. Unlike the m_0 in (12) and (13), this inordinately small time constant can be accounted for: it is due to the large number ($N/V \sim 10^{97}$ per cm^3) of agitated PPs in the PV contributing simultaneously to the ZP field fluctuations described by (8). It is noted in passing that the size of the parton ($\sim r_*$) is not connected to its mass m_0 by the usual Compton relation (i.e., $r_* m_0 c^2 \neq e_*^2$) as is the case for the PP ($r_* m_* c^2 = e_*^2$).

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