The Crothers Metrics and the Black Hole Metric As Viewed From The Planck Vacuum Perspective[1]

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The Crothers metrics [1] represent a denumerable infinity of Schwarzschild-like metrics that describe the curvature a point mass exerts on spacetime. In contrast to the black hole metric, these metrics are continuous down to, but not including, the point mass itself. The present paper compares the Corthers and black-hole metrics in terms of the relative curvature the point mass exerts on the Planck vacuum (PV) negative-energy state [2], and connects the general-relativity calculations to the PV constants. The results cast serious doubt on the mathematical and physical validity of the black hole model.

1 Planck Vacuum Constants

The concept of the universe envisioned here consists of free space, plus the visible universe, plus the electromagnetic component of the quantum vacuum [3] [4], and an invisible (i.e., not directly observable) vacuum state from which the universe emerges. The free-space electric and magnetic permittivities

$$\epsilon = \frac{1}{\mu} = \frac{e_*^2}{r_* m_* c^2} = 1 \tag{1}$$

imply that that supporting vacuum state is the PV [2], where e_*, m_*, r_*, and c are respectively the bare charge, the Planck mass, the Planck-particle Compton radius, and the speed of light. In addition, the two forces

$$\frac{e_*^2}{r_*^2} \qquad \text{and} \qquad \frac{c^4}{G} = \frac{m_* c^2}{r_*} \tag{2}$$

represent the maximum polarization and curvature forces sustainable by the PV, where G is Newton's gravitational constant.

The force difference

$$\frac{e_*^2}{r^2} - \frac{m_* c^2}{r} = 0 \tag{3}$$

vanishes at $r = r_*$ and leads to the Compton relation

$$r_* m_* c^2 = e_*^2 = c\hbar \tag{4}$$

associated with the PV, where $\hbar$ is the (reduced) Planck constant. The relationships in (2) and (4) between the *primary* PV constants (e_*, m_*, r_*) are used repeatedly in what follows to rid the general-relativity equations of the *secondary* constant G.

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2 Crothers Metrics

The general solution to the Einstein field equations [1] for a point mass m at $r = 0$ consists of the infinite collection ($n = 1, 2, 3, \cdots$) of Schwarzschild-like metrics that are *non-singular* for all $r > 0$:

$$ds^2 = (1 - \alpha/R_n)\, c^2 dt^2 - \frac{(r/R_n)^{2n-2}\, dr^2}{1 - \alpha/R_n} - R_n^2\, (d\theta^2 + \sin^2\theta\, d\phi^2) \tag{5}$$

where

$$\alpha = 2\,\frac{mG}{c^2} = 2\,\frac{mc^2}{m_* c^2/r_*} = 2rn_r \tag{6}$$

$$R_n = (r^n + \alpha^n)^{1/n} = r(1 + 2^n n_r^n)^{1/n} = \alpha(1 + 1/2^n n_r^n)^{1/n} \tag{7}$$

and, from the PV theory [5]

$$n_r \equiv \frac{mc^2/r}{m_* c^2/r_*} \tag{8}$$

where r is the *coordinate* radius from the point mass to the field point of interest. The n-ratio n_r is the relative stress the point mass exerts on the PV, its allowable range being $0 \leq n_r < 1$ for which

$$\frac{\alpha}{R_n} < 1\,. \tag{9}$$

From the final expression in (7) it is clear that α/R_n vanishes as r increases without limit. The original Schwarzschild metric [6] corresponds to $n = 3$.

The n-ratio n_r is a direct measure of spacetime flatness, where spacetime is flat for $n_r = 0$ ($r \to \infty$). Asymptotic flatness [7, p.55] thus corresponds to

$$n_r \approx 0 \qquad \text{or} \qquad \frac{mc^2}{r} \ll \frac{m_* c^2}{r_*} \tag{10}$$

$m_* c^2/r_*$ being the maximum curvature force. For a white dwarf and a neutron star, e.g., the n-ratios at the stars' surfaces are $n_r \sim 0.0002$ and $n_r \sim 0.2$ respectively.

The magnitude of the relative coordinate velocity of a photon approaching or leaving the point mass in a radial direction is calculated from the metric coefficients in (5) by setting $ds = 0$, $d\theta = 0$, $d\phi = 0$, and leads to

$$\beta_n(n_r) = \frac{dr}{c\,dt} = \left(\frac{g_{00}}{-g_{11}}\right)^{1/2} = \left[\frac{1 - \alpha/R_n}{(r/R_n)^{2n-2}/(1 - \alpha/R_n)}\right]^{1/2} \tag{11}$$

$$= (1 + 2^n n_r^n)^{(1-1/n)} \left(1 - \frac{2n_r}{(1 + 2^n n_r^n)^{1/n}}\right) \tag{12}$$

whose plot as a function of n_r in Figure 1 shows β_n's behavior as n increases from 1 to 20. The vertical and horizontal axes run from 0 to 1. The limiting case as n increases without limit is

$$\beta_\infty(n_r) = \begin{cases} 1 - 2n_r, & 0 \le n_r \le 0.5 \\ 0, & 0.5 \le n_r < 1\,. \end{cases} \tag{13}$$

That is, the photon does not propagate ($\beta_\infty(n_r) = 0$) in the region $0.5 \le n_r < 1$ for the limiting case.

3 Black Hole Metric

Assuming that

$$R_n = r \tag{14}$$

in (5) leads to the black hole metric [7, p.15]

$$\begin{aligned} ds^2 &= (1 - r_s/r)\, c^2 dt^2 - \frac{dr^2}{(1 - r_s/r)} - r^2\,(d\theta^2 + \sin^2\theta\, d\phi^2) \\ &= (1 - 2n_r)\, c^2 dt^2 - \frac{dr^2}{(1 - 2n_r)} - r^2\,(d\theta^2 + \sin^2\theta\, d\phi^2) \end{aligned} \tag{15}$$

where the so-called Schwarzschild radius is

$$r_s = 2\,\frac{mG}{c^2} = 2\,\frac{mc^2}{m_* c^2/r_*} = 2rn_r\,. \tag{16}$$

The interior ($r < r_s$) is called the black hole. Within this black hole is the naked singularity at the coordinate radius $r = 0$ where the black-hole mass is assumed to reside—hiding this singularity is the event-horizon (at $n_r = 0.5$) with the Schwarzschild radius $r = r_s$.

The singularity in the metric coefficient $g_{11} = -1/(1-2n_r)$ is only apparent and can be removed. Nevertheless it proves interesting to formally calculate the relative velocity of a radially directed photon from (15) by setting $ds = 0$, $d\theta = 0$, and $d\phi = 0$:

$$\beta(n_r) = \frac{dr}{c\,dt} = \left(\frac{g_{00}}{-g_{11}}\right)^{1/2} = \left[\frac{1 - 2n_r}{1/(1-2n_r)}\right]^{1/2} = |1 - 2n_r| \tag{17}$$

where $\beta(0) = \beta(1) = 1$ and $\beta(0.5) = 0$. Thus the incoming photon velocity decreases from c at infinity to zero at $n_r = 0.5$ only to increase to c again at the position of the point mass! This clear violation of common sense emphasizes the fact that the metric (15) needs to be transformed to another set of coordinates to be more useful.

The proper ("as seen from a great distance") circumference, area, and acceleration at the black hole surface (event horizon) are [7, pp.19,19,43]

$$\text{Circumference} = 4\pi\frac{mG}{c^2} = 2\pi r \cdot 2n_r \tag{18}$$

$$\text{Area} = 16\pi \left(\frac{mG}{c^2}\right)^2 = 4\pi r^2 \cdot (2n_r)^2 \tag{19}$$

and

$$\text{Acceleration} = \frac{mG}{c^2 r^2}\left(1 - 2\frac{mG}{cr}\right)^{-1/2} = \frac{c^2}{r}\frac{n_r}{(1-2n_r)^{1/2}} \tag{20}$$

where $r = r_s$ and $n_r = 0.5$ in (18)–(20).

4 Comments and Conclusions

For the reader needing more evidence that the equations of general relativity are tied to the PV, it may help to take a look at how the Kerr-Newman black-hole area A [7, p.105] [8] for a spinning and charged mass point is normalized in the PV theory. Before replacing the secondary constant G in the area equation by the primary constants (e_*, m_*, r_*) from the PV theory, the equation looks like

$$A = \frac{4\pi G}{c^4}\left[2m^2G - Q^2 + 2(m^4G^2 - c^2J^2 - m^2Q^2G)^{1/2}\right] \tag{21}$$

where Q and J are the charge and angular momentum of the mass m.

By using the relations in equations (2) and (4), it is straightforward to transform (21) into the following equation

$$\frac{A}{4\pi r_*^2} = 2\left(\frac{m}{m_*}\right)^2 - \left(\frac{Q}{e_*}\right)^2 + 2\left[\left(\frac{m}{m_*}\right)^4 - \left(\frac{J}{r_* m_* c}\right)^2 - \left(\frac{m}{m_*}\right)^2\left(\frac{Q}{e_*}\right)^2\right]^{1/2} \tag{22}$$

where each ratio in the equation is dimensionless. In addition, all of the terms are properly normalized; the area A by the area $4\pi r_*^2$, the angular momentum J by the angular momentum $r_* m_* c$, and so forth. So ridding (21) of G connects the general-relativity calculations to the PV normalizing constants in a dramatic way. Equation (22) reduces as it should to equation (19) when $Q = 0$ and $J = 0$.

Figure 1 shows that photon velocities decrease monotonically in the range $0.5 \leq n_r < 1$ as the series index n increases, vanishing completely as n increases without limit and shutting off that range from photon propagation. Furthermore, there is a discontinuity of $+2$ in the slope of this limiting-velocity curve as n_r increases from 0.5^- to 0.5^+. Therefor, as there is nothing in the physics of the Crothers metrics to suggest such a discontinuity, the '$n_r \to \infty$' curve should be discarded.

The appearance of the "black hole" range described in the previous paragraph is somewhat analogous to the black-hole-model case for the same region, although that is where the similarity ends. In the black hole model [7, p.48] the event horizon acts as a one-way "membrane" through which exterior photons pass on their way to the naked singularity at $r = 0$.

The black hole model plays a central role in many important astrophysical and cosmological calculations [7, Sec. 1.2] and thus its mathematical validity should be of serious concern. In particular the singularity-introducing substitution

$$R_n = (r^n + \alpha^n)^{1/n} \qquad \longrightarrow \qquad R_n = r \tag{14}$$

used in (5) to arrive at the black hole metric in (15) is troubling and renders the entire black hole model untenable.

References

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[2] Daywitt W.C.: The Planck vacuum. *Progress in Physics*, 2009, v. 1, 20.

[3] Daywitt W.C.: The source of the quantum vacuum. *Progress in Physics*, 2009, v. 1, 27.

[4] Milonni P.W. *The Quantum Vacuum—an Introduction to Quantum Electrodynamics*. Academic Press, New York, 1994.

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[6] Schwarzschild K.: On the gravitational field of a mass point according to Einstein's theory. Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften zu Berlin, Phys.-Math. Klasse, 189(1916). See also arXiv: physics/9905030v1, 1999, May, v. 1, 12.

[7] Raine D., Thomas E.: *Black Holes: An Introduction*. Second Edition, Imperial College Press, London, 2010. The reader should note that m is used in the present paper to denote physical mass—it is not the geometric mass m as used in this reference.

[8] Misner C.W., Thorne K.S., Wheeler J.A.: *Gravitation*. W.H. Freeman and CO., San Francisco, 1973.

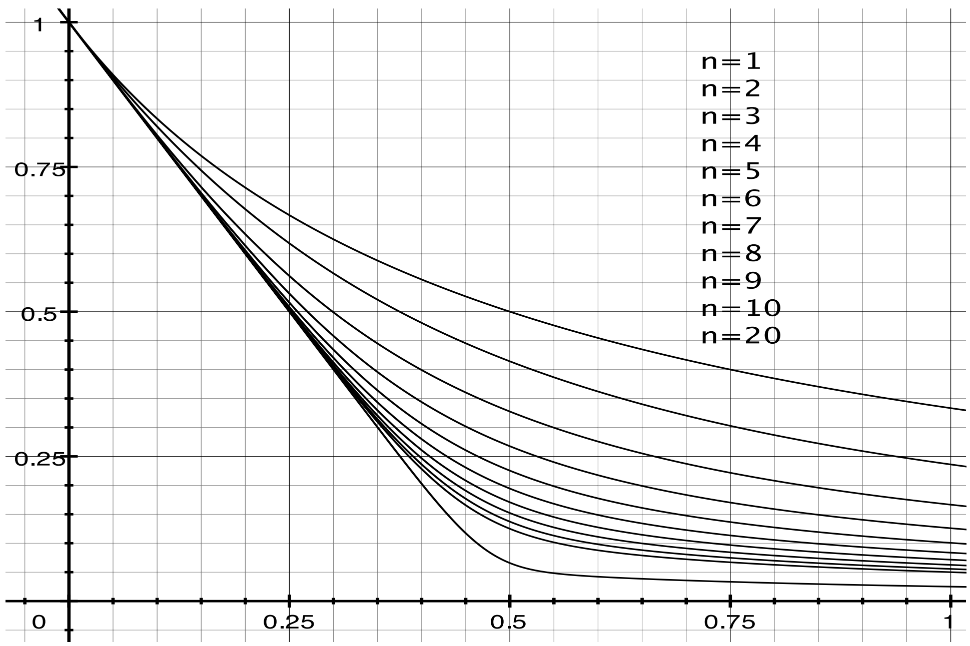

Figure 1. The graph shows the relative photon velocity $\beta_n(n_r)$ plotted as a function of the n-ratio n_r for various indices n. Both axes run from 0 to 1. The limiting case $n \to \infty$ yields $\beta_\infty(n_r) = 1 - 2n_r$ $(0 \leq n_r \leq 0.5)$ and $\beta_\infty(n_r) = 0$ $(0.5 \geq n_r < 1)$. The original Schwarzschild metric corresponds to $n = 3$.