The Source of the Quantum Vacuum

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The quantum vacuum consists of virtual particles randomly appearing and disappearing in free space. Ordinarily the wavenumber (or frequency) spectrum of the zero-point fields for these virtual particles is assumed to be unbounded. The unbounded nature of the spectrum leads in turn to an infinite energy density for the quantum vacuum and an infinite renormalization mass for the free particle. This paper argues that there is a more fundamental vacuum state, the Planck vacuum, from which the quantum vacuum emerges and that the "graininess" of this more fundamental vacuum state truncates the wavenumber spectrum and leads to a finite energy density and a finite renormalization mass.

1 Introduction

The quantum vacuum (QV) [1] consists of virtual particles which are created alone (photons) or in massive particle-antiparticle pairs, both of which are jumping in and out of existence within the constraints of the Heisenberg uncertainty principle ($\triangle E \Delta t \sim \hbar$); i.e., they appear in free space for short periods of time (Δt) depending upon their temporal energy content ($\triangle E$) and then disappear. So the QV is an ever-changing collection of virtual particles which disappear after their short lifetimes Δt to be replaced by new virtual particles that suffer the same fate, the process continuing ad infinitum. The photon component of the QV is referred to here as the electromagnetic vacuum (EV) and the massive-particle component as the massive particle vacuum (MPV).

The quantum fields ascribed to the elementary particles are considered to be the "essential reality" [2] behind the physical universe; i.e., a set of *fields* is the fundamental building block out of which the visible universe is constructed. For example, the vector potential for the quantized electromagnetic field can be expressed as [1, p. 45]

$$\mathbf{A}(\mathbf{r},t) = \sum_{s=1}^{2}\sum_{\mathbf{k}} \left(\frac{2\pi c\hbar}{kV}\right)^{1/2} \times \\ \times \left[a_{\mathbf{k},s}(t)\exp\left(i\mathbf{k}\cdot\mathbf{r}\right) + \mathrm{h.c.}\right]\mathbf{e}_{\mathbf{k},s}\,, \tag{1}$$

where the first sum is over the two polarizations of the field, $k = |\mathbf{k}|$, $V = L^3$ is the box-normalization volume, $a_{\mathbf{k},s}(t)$ is the photon annihilation operator, $h.c.$ stands for the Hermitian conjugate of the first term within the brackets, and $\mathbf{e}_{\mathbf{k},s}$ is the unit polarization vector. This is the quantized vector potential for the EV component of the QV. The vector potential satisfies the periodicity conditions

$$\mathbf{A}(x+L, y+L, z+L, t) = \mathbf{A}(x,y,z,t) \tag{2}$$

or equivalently

$$\mathbf{k} = (k_x, k_y, k_z) = (2\pi/L)(n_x, n_y, n_z)\,, \tag{3}$$

where the n_i can assume any positive or negative integer or zero. Since the Planck constant $\hbar$ is considered to be a primary constant, the field in (1) is a fundamental field that is not derivable from some other source (e.g. a collection of charged particles). This paper argues that $\hbar$ is not a primary constant and thus that there is a more fundamental reality behind the quantum fields.

The most glaring characteristic of the EV (and similarly the MPV) is that its zero-point (ZP) energy [1, p. 49]

$$\sum_{s=1}^{2}\sum_{\mathbf{k}}\frac{\hbar\omega_k}{2} = c\hbar\sum_{\mathbf{k},s}\frac{k}{2} \tag{4}$$

is infinite because of the unbounded nature of the $\mathbf{k}$ ($|k_i| < \infty$) in (3). The sum on the right side of the equal sign is an abbreviation for the double sum on the left and $\omega_k = ck$. Using the well-known replacement

$$\sum_{\mathbf{k},s} \longrightarrow \sum_{s}\left(\frac{L}{2\pi}\right)^3 \int d^3k = \frac{V}{8\pi^3}\sum_{s}\int d^3k \tag{5}$$

in (4) leads to the EV energy density

$$\frac{c\hbar}{V}\sum_{\mathbf{k},s}\frac{k}{2} = \frac{c\hbar}{2\pi^2}\int_0^\infty k^3\,dk = \infty\,, \tag{6}$$

where the infinite upper limit on the integral is due to the unbounded $\mathbf{k}$ in (3).

The present paper does two things: it identifies a charged vacuum state (the PV [3]) as the source of the QV; and calculates a cutoff wavenumber (based on an earlier independent calculation [4]) for the integral in (6). The PV model is presented in the Section 2. In a stochastic-electrodynamic (SED) calculation [4] Puthoff derives the particle mass, the cutoff wavenumber (in terms of the speed of light, the Planck constant, and Newton's gravitational constant), and the gravitational force. The Puthoff model is reviewed in Section 3 and the resulting cutoff wavenumber changed into a form more useful to the present needs by substituting derived relations [3] for the Planck and gravitational constants.

Section 4 argues that the QV has its source in the PV. It accomplishes this result by comparing the PV and QV energy densities. The reader is asked to excuse the course nature of the comparisons used to make the argument. Section 5 comments on the previous sections and expands the PV theory somewhat.

The de Broglie radius is derived in Appendix A to assist in the calculations of Section 4. The derivation is superficially similar to de Broglie's original derivation [5], but differs essentially in interpretation: here the radius arises from the two-fold perturbation the free particle exerts on the PV.

2 Planck vacuum

The PV [3] is an omnipresent degenerate gas of negative-energy Planck particles (PP) characterized by the triad (e_*, m_*, r_*), where e_*, m_*, and r_* $(\lambda_*/2\pi)$ are the PP charge, mass, and Compton radius respectively. The charge e_* is the bare (true) electronic charge common to all charged elementary particles and is related to the observed electronic charge e through the fine structure constant $\alpha = e^2/e_*^2$ which is a manifestation of the PV polarizability. The PP mass and Compton radius are equal to the Planck mass and length respectively. In addition to the fine structure constant, the particle-PV interaction is the ultimate source of the gravitational $(G = e_*^2/m_*^2)$ and Planck $(\hbar = e_*^2/c)$ constants, and the string of Compton relations relating the PV and its PPs to the observed elementary particles and their bare charge e_*

$$r_* m_* c^2 = \cdots = r_c m c^2 = \cdots = e_*^2\,, \tag{7}$$

where the charged elementary particles are characterized by the triad (e_*, m, r_c), m and r_c being the mass and Compton radius $(\lambda_c/2\pi)$ of the particle. Particle spin is not yet included in the theory. The ZP random motion of the PP charges e_* about their equilibrium positions within the PV, and the PV dynamics, are the source of both the free particles and the QV.

The Compton relations (7) have their origin in the two-fold perturbation of the PV by the free particle which polarizes and "curves" (in a general relativistic sense) the PV. The particle-PV interaction is such that the polarization force (e_*^2/r^2) and the curvature force (mc^2/r) are equal at the Compton radius r_c [3]:

$$\frac{e_*^2}{r^2} = \frac{mc^2}{r} \quad \longrightarrow \quad r_c m c^2 = e_*^2\,, \tag{8}$$

where the second equation can be expressed in its usual form $r_c m c = \hbar$. The requirement that the force equality in (8) hold in any Lorentz frame leads to the momentum $(\widehat{\mathbf{p}} = -i\hbar\nabla)$ and energy $(\widehat{E} = i\hbar\partial/\partial t)$ operators and to the de Broglie radius (Appendix A). The so-called "wave-particle duality" of the particle follows from the coupling of the free particle to the (almost) continuous nature of the PV whose continuum supports the wave associated with the wave property ascribed to the particle.

3 Puthoff model

One of the charges in the product e_*^2 terminating the chain of Compton relations (7) belongs to the free particle while the other represents the magnitude of the PP charges making up the PV. The fact that the bare charge is common to all the charged elementary particles depicted by (7) suggests that perhaps e_* is massless, and that the mass m in the particle triad (e_*, m, r_c) results from some reaction of the charge to the ZP fields. In a seminal paper [4] Puthoff, in effect, exploits the idea of a massless charge to derive the particle mass, the wavenumber k_{c*} truncating the spectrum of the ZP fields, and the Newtonian gravitational force. This section reviews Puthoff's SED calculations and casts them into a form convenient to the present needs. Some minor license is taken by the present author in the interpretation behind equations (12) and (13) concerning the constant A.

The Puthoff model starts with a particle equation of motion (EoM) for the mass m_0

$$m_0 \ddot{\mathbf{r}} = e_* \mathbf{E}_{zp}\,, \tag{9}$$

where m_0, considered to be some function of the actual particle mass m, is eliminated from (9) by substituting the damping constant

$$\Gamma = \frac{2e_*^2}{3c^3 m_0} \tag{10}$$

and the electric dipole moment $\mathbf{p} = e_*\mathbf{r}$, where $\mathbf{r}$ represents the random excursions of the charge about its average position at $\langle \mathbf{r} \rangle = 0$. The force driving the particle charge is $e_*\mathbf{E}_{zp}$, where $\mathbf{E}_{\mathrm{zp}}$ is the ZP electric field (B5). Equation (9) then becomes

$$\ddot{\mathbf{p}} = \frac{3c^3\Gamma}{2}\,\mathbf{E}_{\mathrm{zp}}\,, \tag{11}$$

which is an EoM for the charge that, *from here on*, is considered to be a new equation in two unknowns, Γ and the cutoff wavenumber k_{c*}. The mass m of the particle is then defined via the stochastic kinetic energy of the charge whatever that may be. A reasonable guess is the kinetic energy of the discarded mass m_0

$$mc^2 \sim \left\langle \frac{m_0 \dot{\mathbf{r}}^2}{2} \right\rangle = \frac{\langle \dot{\mathbf{p}}_2^2 \rangle}{3c^3\Gamma} \tag{12}$$

realizing that, at best, this choice is only a guide to predicting what parameters to include in the mass definition. The dipole variation $\dot{\mathbf{p}}_2$ is explained below. The simplest definition for the mass is then

$$m \equiv \frac{1}{c^2}\,\frac{A\,\langle \dot{\mathbf{p}}_2^2 \rangle}{3c^3\Gamma}\,, \tag{13}$$

where A is a constant to be determined, along with Γ and k_{c*}, from a set of three experimental constraints.

The three constraints used to determine the three constants Γ, k_{c*}, and A are: 1) the observed mass m of the particle; 2) the perturbed spectral energy density of the EV caused by radiation due to the random accelerations experienced by the particle charge e_* as it is driven by the random force

$e_*\mathbf{E}_{zp}$; and 3) Newton's gravitational attraction between two particles of mass m.

The dipole moment $\mathbf{p}$ in (11) can be readily determined using the Fourier expansions [6]

$$\mathbf{p}(t) = \int_{-\infty}^{\infty} \widetilde{\mathbf{p}}(\Omega) \exp(-i\Omega t)\, d\Omega/(2\pi)^{1/2} \tag{14}$$

and

$$\mathbf{E}_{zp}(\mathbf{r}, t) = \int_{-\infty}^{\infty} \widetilde{\mathbf{E}}_{zp}(\Omega) \exp(-i\Omega t)\, d\Omega/(2\pi)^{1/2}, \tag{15}$$

where $\widetilde{\mathbf{p}}(\Omega)$ and $\widetilde{\mathbf{E}}_{zp}(\Omega)$ are the Fourier transforms of the dipole moment vector $\mathbf{p}$ and the field $\mathbf{E}_{zp}$ respectively.

The mass of the particle is defined via the planar motion of the charge normal to the instantaneous propagation vector $\mathbf{k}$ in (B5) and results in (Appendix B)

$$\left\langle \dot{\mathbf{p}}_2^2 \right\rangle = 2\left\langle (\widehat{\mathbf{x}} \cdot \dot{\mathbf{p}})^2 \right\rangle = \frac{3\hbar c^5 \Gamma^2 k_{c*}^2}{2\pi}\,, \tag{16}$$

where $\widehat{\mathbf{x}}$ is a unit vector in some arbitrary x-direction and the factor 2 accounts for the 2-dimensional planar motion. When the average (16) is inserted into (13), the constant

$$\Gamma = \frac{2\pi m}{A\hbar k_{c*}^2} \tag{17}$$

emerges in terms of the two as yet unknown constants A and k_{c*}.

Acceleration of the free bare charge e_* by $\mathbf{E}_{zp}$ generates electric and magnetic fields that perturb the spectral energy density of the EV with which $\mathbf{E}_{zp}$ is associated. The corresponding average density perturbation is [4]

$$\overline{\Delta\rho'(k)} = \frac{\hbar c^3 \Gamma^2 k}{2\pi^2 R^4} = \frac{2m^2 c^3 k}{A^2 \hbar k_{c*}^4 R^4}\,, \tag{18}$$

where (17) is used to obtain the final expression, and where R is the radius from the average position of the charge to the field point of interest. An alternative expression for the spectral energy perturbation

$$\Delta\rho(k) = \frac{\hbar k}{2\pi^2 c^3} \left(\frac{mG}{R^2}\right)^2 \tag{19}$$

is calculated [4] from the spacetime properties of an accelerated reference frame undergoing hyperbolic motion, and the equivalence principle from General Relativity. Since the two perturbations (18) and (19) must have the same magnitude, equating the two leads to the cutoff wavenumber

$$k_{c*} = \left(\frac{2\pi c^3}{A\hbar G}\right)^{1/2}, \tag{20}$$

where G is Newton's gravitational constant.

The final unknown constant A in (20) is determined from the gravitational attraction between two particles of mass m calculated [4] using their dipole fields and coupled EoMs, resulting in Newton's gravitational equation

$$F = -\frac{\hbar c^3 \Gamma^2 k_{c*}^2}{\pi R^2} = -\frac{2m^2 G}{A R^2}\,, \tag{21}$$

where (17) and (20) are used to obtain the final expression. Clearly $A = 2$ for the correct gravitational attraction, yielding from (20) and (17)

$$k_{c*} = \left(\frac{\pi c^3}{\hbar G}\right)^{1/2} \left[= \frac{\pi^{1/2}}{r_*}\right] \tag{22}$$

and

$$\Gamma = \frac{\pi m}{\hbar k_{c*}^2} = \frac{mG}{c^3} \left[= \left(\frac{r_*}{r_c}\right) \frac{r_*}{c}\right] \tag{23}$$

for the other two constants. The expressions in the brackets of (22) and (23) are obtained by substituting the PV expressions for the gravitational constant ($G = e_*^2/m_*^2$), the Planck constant ($\hbar = e_*^2/c$), and the Compton relation in (8). The bracket in (22) shows, as expected, that the cutoff wavenumber in (B5) is proportional to the reciprocal of the Planck length r_* (roughly the distance between the PPs making up the PV). The bracket in (23) shows the damping constant Γ to be very small, orders of magnitude smaller than the Planck time r_*/c. The smallness of this constant is due to the almost infinite number ($\sim 10^{99}$ per cm^3) of agitated PPs in the PV contributing simultaneously to the ZP field fluctuations.

An aside: zitterbewegung

SED associates the *zitterbewegung* with the EV [7, p. 396], i.e. with the ZP electric and magnetic fields. In effect then SED treats the EV and the MPV as the same vacuum while the PV model distinguishes between these two vacuum states. Taking place within the Compton radius r_c of the particle, the particle *zitterbewegung* can be viewed [1, p. 323] as an "exchange scattering" between the free particle and the MPV on a time scale of about $r_c/2c$, or a frequency around $2c/r_c$. The question of how the particle mass derived from the averaging process in (13) can be effected with the charge appearing and disappearing from the MPV at such a high frequency naturally arises. For this averaging process to work, the frequency of the averaging must be significantly higher than the *zitterbewegung* frequency. This requirement is easily fulfilled since $ck_{c*} \gg 2c/r_c$. To see that the averaging frequency is approximately equal to the cutoff frequency ck_{c*} one needs only consider the details of the average $\left\langle (\widehat{\mathbf{x}} \cdot \dot{\mathbf{p}})^2 \right\rangle$ in (13) which involves the integral $\int_0^{k_{c*}} k\, dk \approx \int_0^{10^{33}} k\, dk$. Ninety-nine percent of the averaging takes place within the last decade of the integral from 10^{32} to 10^{33} (the corresponding frequency ck in this range being well beyond the Compton frequency c/r_c of any of the observed elementary particles), showing that the effective averaging frequency is close to ck_{c*}.

4 EV and MPV with truncated spectra

The non-relativistic self force acting on the free charge discussed in the previous section can be expressed as [1, p. 487]

$$e_* \mathbf{E}_{\text{self}} = \frac{2e_*^2}{3c^3} \frac{d\ddot{\mathbf{r}}}{dt} - \ddot{\mathbf{r}}\, \delta m \tag{24}$$

where the radiation reaction force is the first term and the renormalization mass is

$$\delta m = \frac{4e_*^2}{3\pi c^2} \int_0^{k_{c*}} dk \tag{25}$$

assumed here to have its wavenumber spectrum truncated at k_{c*}. An infinite upper limit to the integral corresponds to the box normalization applied in Section 1 to equation (3) where $|n_i| < \infty$ is unbounded. However, if the normal mode functions of the ZP quantum field are assumed to be real waves generated by the collection of PPs within the PV, then the number of modes n_i along the side of the box of length L is bounded and obeys the inequality $|n_i| \leqslant L/2\sqrt{\pi}\, r_*$, where r_* is roughly the separation of the PPs within the PV. Thus the cutoff wavenumber from the previous section ($k_{c*} = \sqrt{\pi}/r_*$) that corresponds to this n_i replaces the infinite upper limit ordinarily assumed for (25). So it is the "graininess" ($r_* \neq 0$) associated with the minimum separation r_* of the PPs in the PV that leads to a bounded k_i and n_i for (3), and which is thus responsible for the finite renormalization mass (25) and the finite energy densities calculated below.

Electromagnetic vacuum

Combining (4) and (5) with a spectrum truncated at k_{c*} leads to the EV energy density [1, p. 49]

$$\frac{c\hbar}{V} \sum_{\mathbf{k},s} \frac{k}{2} = \frac{2c\hbar}{8\pi^3} \int d^3k\, \frac{k}{2} = \frac{c\hbar}{4\pi^3} \int d\Omega_k \int_0^{k_{c*}} dk\, k^2 \frac{k}{2} =$$
$$= \frac{4\pi c\hbar}{4\pi^3} \int_0^{k_{c*}} dk\, k^2 \frac{k}{2} = \frac{c\hbar}{2\pi^2} \frac{k_{c*}^4}{4} = \frac{c\hbar}{8r_*^4} = \frac{1}{8} \frac{e_*^2/r_*}{r_*^3}, \tag{26}$$

where the 2 in front of the triple integral comes from the sum over $s = 1, 2$; and where $k_{c*} = \sqrt{\pi}/r_*$ and $c\hbar = e_*^2$ are used to obtain the final two expressions. If the energy density of the PV (excluding the stochastic kinetic energy of its PPs) is assumed to be roughly half electromagnetic energy ($\sim e_*^2/r_*$) and half mass energy ($\sim m_* c^2$), then

$$\frac{e_*^2/r_* + m_* c^2}{r_*^3} = 2\frac{e_*^2/r_*}{r_*^3} \tag{27}$$

is a rough estimate of this energy density. Thus the energy density (26) of the EV (the virtual-photon component of the QV) is at most one sixteenth ($1/16$) the energy density (27) of the PV. Although this estimate leaves much to be desired, it at least shows the EV energy density to be less than the PV energy density which must be the case if the PV is the source of the EV.

Massive particle vacuum

The energy density of the ZP Klein-Gordon field is [1, p. 342]

$$\frac{\langle 0|H|0\rangle}{V} = \frac{1}{2V} \int d^3k E_k\, \delta^3(0) =$$
$$= \frac{\delta^3(0)}{2V} \int d\Omega_k \int_0^{k_{c*}} dk\, k^2 E_k = \frac{1}{4\pi^2} \int_0^{k_{c*}} k^2 E_k\, dk =$$
$$= \frac{e_*^2}{4\pi^2} \int_0^{k_{c*}} k^2 \left(k_c^2 + k^2\right)^{1/2} dk\,, \tag{28}$$

where $\delta^3(0) = V/8\pi^3$ is used to eliminate $\delta^3(0)$ and $E_k = e_*^2 \sqrt{k_c^2 + k^2}$ comes from (A5). Equation (28) leads to

$$\frac{\langle 0|H|0\rangle}{V} = \frac{e_*^2 k_c}{4\pi^2} \int_0^{k_{c*}} k^2 \left[1 + \left(\frac{k}{k_c}\right)^2\right]^{1/2} dk =$$
$$= \frac{e_*^2/r_c}{4\pi^2} \int_0^{k_{c*}} k^2 \left[1 + (r_c k)^2\right]^{1/2} dk =$$
$$= \frac{e_*^2/r_c}{4\pi^2 r_c^3} \int_0^{r_c k_{c*}} x^2 (1 + x^2)^{1/2} dx =$$
$$= \frac{1}{16} \frac{e_*^2/r_*}{r_*^3} \left(1 + \frac{r_*^2}{\pi r_c^2} + \cdots\right) \approx \frac{1}{16} \frac{e_*^2/r_*}{r_*^3}, \tag{29}$$

where $k_c = 1/r_c$ is used in the first line. The final integral is easily integrated [8] and leads to the expansion in the second-to-last expression. The final expression follows from the fact that the second ($r_*^2/\pi r_c^2 \sim 10^{-40}$) and higher-order terms in the expansion are vanishingly small (the ratio $r_*/r_c \sim 10^{-20}$ is used as a rough average for the ratio of the Compton radii of the PP and the observed elementary particles). So the energy density in (29) is one thirty-second ($1/32$) of the PV energy density in (27).

The k^2 term under the radical sign in (28) corresponds to the squared momentum of the massive virtual particles contributing to the average vacuum density described by (28). The second term in the large parenthesis of (29) is approximately the relative contribution of the virtual-particle mass to the overall energy density as compared to the coefficient in front of the parenthesis which represents the energy density of the virtual-particle kinetic energy. Thus the kinetic energy of the virtual particles in the MPV dominates their mass energy by a factor of about 10^{40}.

5 Conclusion and comments

The conclusion that the PV is the source of the quantum fields is based on the fact that $\hbar\ (= e_*^2/c)$ is a secondary constant, where one of the e_*s in the product e_*^2 is the particle charge and the other is the charge on the PPs making up the PV; and that the amplitude factor A_k in the ZP electric field (B5) is proportional to the charge on the PPs in the PV. The ubiquitous nature of $\hbar\omega = e_*^2 k$ in the quantum field equations, whether k is an electromagnetic wavenumber or a de Broglie wavenumber, further supports the conclusion.

The Compton relations (7) and the Puthoff model in Section 3 both suggest that the particle charge e_* is massless. To be self-consistent and consistent with the Puthoff model, the PV model for the Compton relations must assume that the Compton radius $r_c = r_c(m) = e_*^2/mc^2$ is larger than the structural extent of the particle and the random excursions of the charge leading to the mass (13).

The PV theory has progressed to this point without addressing particle spin — its success without spin suggesting perhaps that spin is an acquired, rather than an intrinsic, property of the particle. A circularly polarized ZP electric field may, in addition to generating the mass in (13), generate an effective spin in the particle. This conclusion follows from a SED spin model [7, p. 261] that uses a circularly polarized ZP field in the modeling process — in order to avoid too much speculation though, one question left unexplored in this spin model is how the ZP field acquires the circular polarization needed to drive the particle's spin. Perhaps the ZP field acquires its circular polarization when the magnetic field probing the particle (a laboratory field or the field of an atomic nucleus) induces a circulation within the otherwise random motion of the PP charges in the PV, these charges then feeding a circular polarization back into the ZP electric field $\mathbf{E}_{\mathrm{zp}}$ of the EV, thus leading to the particle spin.

Appendix A de Broglie radius

A charged particle exerts two distorting forces on the collection of PPs constituting the PV [3], the polarization force e_*^2/r^2 and the curvature force mc^2/r. The equality of the two force magnitudes at the Compton radius r_c in (8) is assumed to be a fundamental property of the particle-PV interaction. The vanishing of the force difference $e_*^2/r_c^2 - mc^2/r_c = 0$ at the Compton radius can be expressed as a vanishing tensor 4-force [9] difference. In the primed rest frame of the particle where these static forces apply, this force difference $\Delta F'_\mu$ is ($\mu = 1, 2, 3, 4$)

$$\Delta F'_\mu = \left[\mathbf{0},\ i\left(\frac{e_*^2}{r_c^2} - \frac{mc^2}{r_c}\right)\right] = [0,\ 0,\ 0,\ i\,0]\,, \tag{A1}$$

where $i = \sqrt{-1}$. Thus the vanishing of the 4-force component $\Delta F'_4 = 0$ in (A1) is the source of the Compton relation in (8) which can be expressed in the form $mc^2 = e_*^2/r_c = (e_*^2/c)(c/r_c) = \hbar\omega_c$, where $\omega_c \equiv c/r_c = mc^2/\hbar$ is the Compton frequency corresponding to the Compton radius r_c.

The 4-force difference in the laboratory frame, that is $\Delta F_\mu = a_{\mu\nu}\Delta F'_\nu = 0_\mu$, follows from its tensor nature and the Lorentz transformation $x_\mu = a_{\mu\nu}\, x'_\nu$ [9], where $x_\mu = (x, y, z, ict)$,

$$a_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \gamma & -i\beta\gamma \\ 0 & 0 & i\beta\gamma & \gamma \end{pmatrix} \tag{A2}$$

and $\mu, \nu = 1, 2, 3, 4$. Thus (A1) becomes

$$\begin{aligned} \Delta F_\mu &= \left[0,\ 0,\ \beta\gamma\left(\frac{e_*^2}{r_c^2} - \frac{mc^2}{r_c}\right),\ i\gamma\left(\frac{e_*^2}{r_c^2} - \frac{mc^2}{r_c}\right)\right] = \\ &= \left[0,\ 0,\ \left(\frac{e_*^2}{\beta\gamma r_d^2} - \frac{mc^2}{r_d}\right),\ i\left(\frac{e_*^2}{\gamma r_L^2} - \frac{mc^2}{r_L}\right)\right] = \\ &= [0,\ 0,\ 0,\ i\,0] \end{aligned} \tag{A3}$$

in the laboratory frame. The equation $\Delta F_3 = 0$ from the final two brackets yields the de Broglie relation

$$p = \frac{e_*^2/c}{r_d} = \frac{\hbar}{r_d} = \hbar k_d \tag{A4}$$

where $p = m\gamma v$ is the relativistic particle momentum, $r_d \equiv r_c/\beta\gamma$ is the de Broglie radius, and $k_d = 1/r_d$ is the de Broglie wavenumber.

Using (8) and (A4), the relativistic particle energy can be expressed as

$$\begin{aligned} E_{k_d} &= \left(m^2c^4 + c^2p^2\right)^{1/2} = \\ &= \left(e_*^4 k_c^2 + c^2\hbar^2 k_d^2\right)^{1/2} = e_*^2\left(k_c^2 + k_d^2\right)^{1/2}, \end{aligned} \tag{A5}$$

where $mc^2 = e_*^2/r_c$, $k_c = 1/r_c$, and $c\hbar = e_*^2$ are used to obtain the final two expressions.

The equation $\Delta F_4 = 0$ from (A3) leads to the relation $p = \hbar/r_L$, where $r_L \equiv r_c/\gamma$ is the length-contracted r_c in the ict direction. The Synge primitive quantization of flat spacetime [10] is equivalent to the force-difference transformation in (A3): the ray trajectory of the particle in spacetime is divided (quantized) into equal lengths of magnitude $\lambda_c = 2\pi r_c$ (this projects back on the "ict" axis as $\lambda_L = 2\pi r_L$); and the de Broglie wavelength calculated from the corresponding spacetime geometry. Thus the development in the previous paragraphs provides a physical explanation for Synge's spacetime quantization in terms of the two perturbations e_*^2/r^2 and mc^2/r the free particle exerts on the PV.

Appendix B Charge EoM with the self force

Combining (24) and (25) leads to the charge's self force

$$e_*\mathbf{E}_{\mathrm{self}} = \frac{2e_*^2}{3c^3}\left(\frac{d\ddot{\mathbf{r}}}{dt} - \omega'_*\ddot{\mathbf{r}}\right) \tag{B1}$$

with $\omega'_* \equiv 2c/\sqrt{\pi}\, r_*$. Adding (B1) to the right side of (9) then yields the x-component of the charge's acceleration corresponding to (11):

$$\ddot{x} = \Gamma\left(\frac{d\ddot{x}}{dt} - \omega'_*\ddot{x}\right) + \frac{3c^3\Gamma}{2e_*}\,\widehat{\mathbf{x}}\cdot\mathbf{E}_{\mathrm{zp}} \tag{B2}$$

which can be solved by the Fourier expansions

$$x(t) = \int_{-\infty}^{\infty} \widetilde{x}(\Omega)\exp\left(-i\Omega t\right) d\Omega/(2\pi)^{1/2} \tag{B3}$$

and

$$E_x(\mathbf{r}, t) = \int_{-\infty}^{\infty} \widetilde{E}_x(\Omega)\exp\left(-i\Omega t\right) d\Omega/(2\pi)^{1/2} \tag{B4}$$

where $E_x \equiv \widehat{\mathbf{x}}\cdot\mathbf{E}_{\mathrm{zp}}$, and where the ZP electric field $\mathbf{E}_{\mathrm{zp}}$ is assumed to have an upper cutoff wavenumber k_{c*} [4, 3]:

$$\begin{aligned} \mathbf{E}_{zp}(\mathbf{r}, t) = \mathrm{Re}\sum_{\sigma=1}^{2}\int d\Omega_k\int_0^{k_{c*}} dk\, k^2\,\widehat{\mathbf{e}}_\sigma(\mathbf{k})A_k\,\times \\ \times\ \exp\left[\,i\left(\mathbf{k}\cdot\mathbf{r} - \omega t + \Theta_\sigma(\mathbf{k})\right)\right], \end{aligned} \tag{B5}$$

where Re stands for "real part of"; the sum is over the two transverse polarizations of the random field; the first integral is over the solid angle in k-space; $\widehat{\mathbf{e}}_\sigma$ is the unit polarization vector; $A_k = \sqrt{\hbar\omega/2\pi^2} = e_* \sqrt{k/2\pi^2}$ is the amplitude factor which is proportional to the bare charge e_* of the PPs in the PV; $\omega = ck$; and Θ_σ is the random phase that gives $\mathbf{E}_{\mathrm{zp}}$ its stochastic character.

The inverse Fourier transform of E_x from (B4) works out to be

$$\begin{aligned}\widetilde{E}_x(\Omega) = \left(\frac{\pi}{2}\right)^{1/2} \sum_{\sigma=1}^{2} \int d\Omega_k \int_0^{k_{c*}} dk\, k^2\, \widehat{\mathbf{x}} \cdot \widehat{\mathbf{e}}_\sigma(\mathbf{k}) A_k \times \\ \times \big\{\delta(\Omega-\omega) \exp\left[\,i\left(\mathbf{k}\cdot\mathbf{r} + \Theta_\sigma(\mathbf{k})\right)\right] + \\ + \delta(\Omega+\omega) \exp\left[-i\left(\mathbf{k}\cdot\mathbf{r} + \Theta_\sigma(\mathbf{k})\right)\right]\big\}\end{aligned} \tag{B6}$$

in a straightforward manner, where $\delta(\Omega-\omega)$ and $\delta(\Omega+\omega)$ are Dirac delta functions. Equation (B6) is easily checked by inserting it into (B4) and comparing the result with $\widehat{\mathbf{x}} \cdot \mathbf{E}_{\mathrm{zp}}$ from (B5).

Calculating $\ddot{\mathbf{x}}$ and $d\ddot{\mathbf{x}}/dt$ from (B3) and inserting the results, along with (B4), into (B2) leads to the inverse transform

$$\widetilde{x}(\Omega) = -\frac{\left(3c^3\Gamma/2e_*\right)\, \widetilde{E}_x(\Omega)}{(1+\Gamma\omega'_*)\Omega^2 + i\,\Gamma\Omega^3} \tag{B7}$$

for $x(t)$. Then inserting (B7) into (B3) yields

$$\begin{aligned}x(t) = -\left(\frac{3c^3\Gamma}{2e_*}\right) \mathrm{Re} \sum_{\sigma=1}^{2} \int d\Omega_k \int_0^{k_{c*}} dk\, k^2\, \widehat{\mathbf{x}} \cdot \widehat{\mathbf{e}}_\sigma(\mathbf{k}) A_k \times \\ \times \frac{\exp\left[\,i\left(\mathbf{k}\cdot\mathbf{r} - \omega t + \Theta_\sigma(\mathbf{k})\right)\right]}{(1+\Gamma\omega'_*)\omega^2 + i\,\Gamma\omega^3}\end{aligned} \tag{B8}$$

for the random excursions of the charge.

Differentiating (B8) with respect to time while discarding the small Γ terms in the denominator leads to the approximation

$$\begin{aligned}\dot{x}(t) = \left(\frac{3c^3\Gamma}{2e_*}\right) \mathrm{Re} \sum_{\sigma=1}^{2} \int d^3k\, \widehat{\mathbf{x}} \cdot \widehat{\mathbf{e}}_\sigma(\mathbf{k}) \times \\ \times \frac{A_k\, i\omega \exp\left[\,i\left(\mathbf{k}\cdot\mathbf{r} - \omega t + \Theta_\sigma(\mathbf{k})\right)\right]}{\omega^2}\end{aligned} \tag{B9}$$

for the x-directed velocity, from which the dipole average (16)

$$\left\langle \dot{\mathbf{p}}_2^2 \right\rangle = 2\left\langle (\widehat{\mathbf{x}} \cdot \dot{\mathbf{p}})^2 \right\rangle = 2e_*^2 \left\langle \dot{x}^2(t) \right\rangle = \frac{3\hbar c^5 \Gamma^2 k_{c*}^2}{2\pi} \tag{16}$$

follows, where $e_*^2 = c\hbar$ is used to eliminate e_*^2, and

$$\int d^3k \, = \int d\Omega_k \int_0^{k_{c*}} dk\, k^2 \tag{B10}$$

is used to expand the triple integral during the calculation.

Differentiating (B8) twice with respect to the time leads to the dipole acceleration that includes the charge's self force:

$$\begin{aligned}\ddot{\mathbf{p}} = \frac{3}{2}\left(\frac{r_*}{r_c}\right)^2 r_c c^2\, \mathrm{Re} \sum_{\sigma=1}^{2} \int d\Omega_k \int_0^{k_{c*}} dk\, k^2\, \widehat{\mathbf{e}}_\sigma(\mathbf{k}) \times \\ \times \frac{A_k \exp\left[\,i\left(\mathbf{k}\cdot\mathbf{r} - \omega t + \Theta_\sigma(\mathbf{k})\right)\right]}{1 + \Gamma\omega'_* + i\,\Gamma ck}\,,\end{aligned} \tag{B11}$$

which differs from (11) only in denominator on the right side of (B11). The last two terms in the denominator are orders of magnitude smaller than one: $\Gamma\omega'_* < r_*/r_c \sim 10^{-20}$ and $\Gamma ck < \Gamma ck_{c*} = \sqrt{\pi}\, r_*/r_c \sim 10^{-20}$. Thus the charge's self force is not a significant consideration in the definition (13) of the particle's mass.

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References

1. Milonni P.W. The quantum vacuum — an introduction to Quantum Electrodynamics. Academic Press, New York, 1994.
2. Weinberg S. The search for unity — notes for a history of Quantum Field Theory. *Daedalus*, v. 106, 1977, 17.
3. Daywitt W.C. The planck vacuum. *Progress in Physics*, 2009, Jan., v. 1, 20.
4. Puthoff H.E. Gravity as a zero-point-fluctuation force. *Phys. Rev. A*, 1989, v. 39, no. 5, 2333–2342.
5. de Broglie L. Une tentative d'interprétation causale et non linéaire de la mécanique ondulatoire. Gauthier-Villars, Paris, 1956. See Ch. 12.
6. Haisch B., Rueda A., Puthoff H.E. Inertia as a zero-point-field force. *Phys. Rev. A*, v. 49, no. 2, 1994, 678–695. Section III and Appendix A of this reference point out a small problem with the calculations in Section III of ref. [4]. The problem is easily corrected by using Fourier expansions in Section III of [4].
7. de la Peña L., Cetto A.M. The quantum dice — an introduction to Stochastic Electrodynamics. Kluwer Academic Publishers, Boston, 1996.
8. Dwight H.B. Tables of integrals and other mathematical data. The Macmillan Co., 3rd ed., NY, 1947. See equation 232.01.
9. Jackson J.D. Classical Electrodynamics. John Wiley & Sons, 1st ed., 2nd printing, NY, 1962.
10. Synge J.L. Geometrical Mechanics and de Broglie Waves. Cambridge University Press, 1954. See pp.106–107.