The Compton Radius, the de Broglie Radius, the Planck Constant, and the Bohr Orbits

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The Bohr orbits of the hydrogen atom and the Planck constant can be derived classically from the Maxwell equations and the assumption that there is a variation in the electron's velocity about its average value [1]. The resonant nature of the circulating electron and its induced magnetic and Faraday fields prevents a radiative collapse of the electron into the nuclear proton. The derived Planck constant is $h = 2\pi e^2/\alpha c$, where e, α, and c are the electronic charge, the fine structure constant, and the speed of light. The fact that the Planck vacuum (PV) theory [2] derives the same Planck constant independently of the above implies that the two derivations are related. The following highlights that connection.

In the Beckmann derivation [1], the electromagnetic-field mass and the Newtonian mass are assumed to have the same magnitude in which case the electron's average kinetic energy can be expressed as

$$\left(\frac{mv^2}{2}\right)_{\rm em} + \left(\frac{mv^2}{2}\right)_{\rm n} = mv^2 = mv{\cdot}v = mv{\cdot}\lambda\nu = mv\lambda{\cdot}\nu = h\nu \quad (1)$$

where v is the average electron velocity and $v = \lambda\nu$ is a simple kinematic relation expressing the fact that the electron's instantaneous velocity varies periodically at a frequency ν over a path length equal to the wavelength λ. The constant $h\,(= mv\lambda)$ turns out to be the Planck constant.

The Beckmann derivation assumes with Maxwell and those following thereafter that the magnetic and Faraday fields are part of the electron makeup. On the other hand the PV theory assumes that these fields constitute a reaction of the negative-energy PV quasi-continuum to the movement of the massive point charge (the Dirac electron). In its rest frame the electron exerts the two-fold force [3]

$$\frac{e_*^2}{r^2} - \frac{mc^2}{r} \quad (2)$$

on each point r of the PV, where $e_*\ (= e/\sqrt{\alpha})$ is the electron's bare charge, e is the laboratory-observed charge, and m is the electron mass. The vanishing of this composite force at the radius $r = r_c$ leads to

$$r_c mc^2 = e_*^2 = c\hbar = e^2/\alpha, \quad (3)$$

where r_c is the electron's Compton radius and $\hbar$ is the (reduced) Planck constant. From the introductory paragraph and (3), the Beckmann and PV results clearly lead to the same Planck constant $\hbar = e^2/\alpha c = e_*^2/c$.

The Planck constant then is associated only with the bare charge $|e_*|$ and not the electron mass—thus the quantum theory reflects the fact that, although the various elementary particles have different masses, they are associated with only one electric charge.

The expression $mv\lambda = h$ used in (1) to arrive at the total electron kinetic energy is the de Broglie relation expressed in simple, physically intuitive terms: the de Broglie relation yields the product of the electron mass m, its average velocity v, and the path length λ over which its instantaneous velocity varies. The relativistic version of the relationship (which is arrived at in the Appendix by assuming the vanishing of (2) at $r = r_c$ to be a Lorentz invariant constant) is

$$m\gamma v = \frac{\hbar}{r_d} = \frac{\gamma\hbar}{r_c/\beta} = \frac{\gamma h}{\lambda_c/\beta} = \frac{\gamma h}{\lambda} \quad (4)$$

where $m\gamma v$ is the relativistic momentum; and $\lambda = \lambda_c/\beta$, where λ_c is the Compton wavelength $2\pi r_c$. Thus Beckmann's de Broglie relation is in relativistic agreement with the PV result.

The preceding demonstrates that Bohr's introduction of the quantum concept in terms of an ad-hoc Planck constant [4] can be derived from classical electromagnetism and the assumption that the electron interacts with some type of negative-energy vacuum state (the PV in the present case). That the Lorentz transformation can also be derived from the same assumptions is shown in a previous paper [5].

Acknowledgment

The present author's first contact with the late Professor Petr Beckmann was in a course he taught at the University of Colorado (USA) around 1960 on 'Statistical Communication Theory' and later (~circa 1989) in a number of phone conversations concerning his book *Einstein Plus Two* [1]. Much of the work on the PV theory was inspired by Prof. Beckmann's relentless search for the physical truth of things. In addition to authoring a number of interesting books, he founded the scientific journal *Galilean Electrodynamics* and the news letter *Access to Energy* both of which are still active today.

Appendix: de Broglie Radius

The Dirac electron exerts two distortion forces on the collection of Planck particles constituting the degenerate PV, the polarization force e_*^2/r^2 and the curvature force mc^2/r. The equality of the two forces at the electron Compton radius r_c is assumed to be a fundamental property of the electron-PV interaction. The vanishing of the force difference $e_*^2/r_c^2 - mc^2/r_c = 0$ (a Lorentz invariant constant) at the Compton radius can be expressed as a vanishing 4-force difference tensor [6]. In the primed rest frame of the electron, where these static forces apply, this force difference $\Delta F'_\mu$ is

$$\Delta F'_\mu = \left[\mathbf{0}, i\left(\frac{e_*^2}{r_c^2} - \frac{mc^2}{r_c}\right)\right] = [0, 0, 0, i\,0] \tag{A1}$$

where $i = \sqrt{-1}$. Thus the vanishing of the 4-force component $\Delta F'_4 = 0$ in (A1) is the Compton-radius result from (2) and can be expressed in the form $mc^2 = e_*^2/r_c = (e_*^2/c)(c/r_c) = \hbar\omega_c$, where $\omega_c \equiv c/r_c = mc^2/\hbar$ is the corresponding Compton frequency.

The 4-force difference in the laboratory frame, $\Delta F_\mu = a_{\mu\nu}\Delta F'_\nu = 0_\mu$, follows from its tensor nature and the Lorentz transformation $x_\mu = a_{\mu\nu}\, x'_\nu$ [6], where $x_\mu = (x, y, z, ict)$,

$$a_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \gamma & -i\beta\gamma \\ 0 & 0 & i\beta\gamma & \gamma \end{pmatrix} \tag{A2}$$

$\gamma = 1/\sqrt{1-\beta^2}$, and $\mu, \nu = 1, 2, 3, 4$. Thus (A1) becomes

$$\Delta F_\mu = \left[0, 0, \beta\gamma\left(\frac{e_*^2}{r_c^2} - \frac{mc^2}{r_c}\right), i\,\gamma\left(\frac{e_*^2}{r_c^2} - \frac{mc^2}{r_c}\right)\right]$$

$$= \left[0, 0, \left(\frac{e_*^2}{\beta\gamma r_d^2} - \frac{mc^2}{r_d}\right), i\left(\frac{e_*^2}{\gamma r_L^2} - \frac{mc^2}{r_L}\right)\right] = [0, 0, 0, i\,0] \tag{A3}$$

in the laboratory frame. The equation $\Delta F_3 = 0$ from the final two brackets yields the de Broglie relation

$$p = \frac{e_*^2/c}{r_d} = \frac{\hbar}{r_d} \tag{A4}$$

where $p = m\gamma v$ is the relativistic electron momentum and $r_d \equiv r_c/\beta\gamma$ is the de Broglie radius.

The equation $\Delta F_4 = 0$ from (A3) leads to the relation $p = \hbar/r_L$, where $r_L \equiv r_c/\gamma$ is the length-contracted r_c in the *ict* direction. The Synge primitive quantization of flat spacetime [7] is equivalent to the force-difference transformation in (A3): the ray trajectory of the particle in spacetime is divided (quantized) into equal lengths of magnitude $\lambda_c = 2\pi r_c$ (this projects back on the '*ict*' axis as $\lambda_L = 2\pi r_L$); and the de Broglie wavelength calculated from the corresponding spacetime geometry. Thus the development in the previous paragraphs provides a physical explanation for Synge's spacetime quantization in terms of the two perturbations e_*^2/r^2 and mc^2/r the Dirac electron exerts on the PV.

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